DECISION-ANALYTIC APPROACH TO RULE-BASED EXPERT SYSTEM DEVELOPMENT USING GPS AS THE MODEL

THESIS

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Wright-Patterson Air Force Base, Ohio

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GSO/ENS/89D-5	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION School of Engineering 6b. OFFICE SYMBOL (If applicable) AFIT/ENS	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Air Force Institute of Technology (AU) Wright-Patterson AFB, Ohio 45433-6583	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO / WORK UNIT ACCESSION NO

11. TITLE (Include Security Classification)
DECISION-ANALYTIC APPROACH TO RULE-BASED EXPERT SYSTEM DEVELOPMENT USING GPS AS THE MODEL

12. PERSONAL AUTHOR(S)
Grady N. Elliott Jr., Captain U.S. Air Force

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
MS Thesis	FROM ______ TO ______	1989 December	145

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Expert Systems, Decision Theory, Decision Analysis
12	04		Bayesian Decision Theory, Uncertainty Reasoning
12	03		

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Title: Decision-Analytic Approach to Rule-based Expert System Development Using GPS As The Model

Thesis Chairman: Major Bruce W. Morlan
Department Of Operational Sciences

(ABSTRACT ON BACK)

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION
22a. NAME OF RESPONSIBLE INDIVIDUAL Bruce W. Morlan, MS	22b. TELEPHONE (Include Area Code) (513) 255-3362 22c. OFFICE SYMBOL AFIT/ENS

UNCLASSIFIED

Block 19, (Abstract)

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Space Operations

Grady Narvell Elliott Jr., B.S.

Captain, USAF

December, 1989

Approved for public release; distribution unlimited.

Preface

The purpose of this thesis was to determine if decision theory would provide a suitable framework for an expert system reasoning methodology. Particular attention was directed towards how uncertainty and utility assessments could be factored into a decision in a consistent manner. Though the model presented does have some limitations, the approach nonetheless is valid in a number of areas where rational thinking and uncertainty are factored into decision making. Hopefully this research has broken ground for the development of a full-scale expert system developed on the concepts of decision theory.

I wish to thank my thesis advisor Major Bruce (The "More") Morlan for making this both a rewarding and challenging project. With his hands-off management style, he pointed me down the right paths yet allowed me the leeway to stray every now and then in order to enrich my learning experience. Without his patience or confidence this work never would have finished. Further thanks go out to my reader, Major Thomas ("T S") Kelso, for his prudent comments which helped consolidate my ideas and thoughts into a presentable document. Finally, I owe my greatest thanks to my beautiful wife Caroll and my two sons, Eric and Arman, who lived without their husband and father for 18 months while he pursued a personal goal. Your constant love and encouragement made this all possible.

Grady Narvell Elliott Jr.

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Abstract

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I. Introduction

1.1 Background

In the realm of satellite operations, sound and accurate decision making can mean the difference between saving a troubled vehicle or losing it. The recent loss of the Soviet Phobos satellite due to operator error attests to this fact (9:31). As it presently stands, satellite anomaly resolution is a dynamic and often probabilistic process which requires posterior analysis of the available telemetry evidence (E_i) to determine hypotheses (H_i) of the anomaly's cause (8:11). Based on the expected utility of a particular action, a decision is made. Presently this process is performed in a heuristic manner based upon an expert's examination of the given data as well as his beliefs as to what caused the problem. This ability to competently analyze and rectify satellite anomalies is gained from years of hands on experience. As the number of satellites continues to grow, the luxury of this lengthy learning process will disappear.

Because the Global Positioning System (GPS) satellite program is the first such operated by the Air Force, questions have been raised as to how the knowledge base of expertise is going to grow as satellite operators are transferred into and out of the unit. Rule-based expert systems are looked to as a way to capture the growing body of knowledge and expertise. Research by Knue (1986) and Rampino (1987) shows this indeed is a practical and likely option for the Air Force to pursue.

Expert systems are knowledge-based computer programs which behave like experts in chosen domains of application (2:314). They provide a practical means of building automated experts in areas where job excellence requires consistent reasoning and rewards practical experience (29:963). They have proven themselves to be quite beneficial in areas such as medical diagnosis, and geological exploration. In particular, MYCIN, which is used as a consultant to medical personnel, has successfully recommended actions which parallel those of other medical experts (4:589-596). Table 1.1 outlines the different areas rule-based technology has been successfully applied.

Table 1.1. Rule-Based System Applications

Problem	System Functions
Equipment maintenance	Diagnose faults and recommend repairs
Component selection	Elicit requirements and match parts catalog
Computer operation	Analyze requirements; select and operate software
Product configuration	Elicit preferences and identify parts that satisfy constraints
Troubleshooting	Analyze situation, suggest treatments, and prescribe preventative measures
Process control	Spot problematic data and remedy irregularities
Quality assurance	Assess task, propose practices, and enforce requirements

Most expert systems are built upon a series of conditional rules which govern how knowledge is represented and managed. These rules are written in a "If A then B" format. This means that if condition A is true, then conclude B. However, inconsistencies arise when the various systems try to represent incomplete or uncertain knowledge. In MYCIN, an example of this uncertain knowledge would be written into a rule as follows:

IF:

1) The gram stain of the organism is gramneg, and

2) The morphology of the organism is rod, and

3) The aerobicity of the organism is anaerobic

Then: There is suggestive evidence (.6) that the identity of the organism is bactroides (4:71).

How this uncertainty is managed and how rules are combined in order to recommend particular courses of action varies from system to system. This thesis will examine a decision-analytic approach towards managing uncertainty in expert systems.

1.2 Problem Statement

The purpose of this thesis is to demonstrate the applicability of using a Bayesian decision-analytic approach to handle uncertainty and utility in expert system development. The research will focus specifically on how this approach can be used to diagnose anomalous conditions aboard GPS satellites.

1.3 Scope

The scope of this thesis will determine if the GPS contingency procedures and system knowledge can adequately be represented in a decision-analytic format. Particular emphasis will concern the incorporation and management of second-order uncertainty and its effects on anomaly resolution. The thesis is structured as follows. Chapter II is a literature review which examines uncertainty representation in expert sytems. Reasons why decision theory is a viable option will be discussed. Chapter III will outline the methodology and the steps needed to fulfill the research objective. Chapter IV gives background data as to why the GPS program was selected and what successful application of the model in the satellite operations domain could mean. Chapter V discusses the assumptions the decision-analytic model is based on and how it differs from other rule bases in its representation of knowledge. Chapter

VI explains how to apply the method to problems in general while Chapter VII will apply the model to a particular GPS problem . Chapter VIII will summarize conclusions based on results of Chapter VII, and give recommendations as to the overall viability of this approach towards knowledge representation in rule-based expert systems in general.

II. Literature Review

2.1 Introduction

A major point of contention among leaders in the Artificial Intelligence (AI) community is how uncertainty should be expressed and managed in expert systems. There are four major schools of thought, each with differing views concerning uncertain reasoning. These areas are classic probability or Bayesian reasoning, Zadeh's fuzzy set theory, Shortliffe's certainty factors, and Dempster-Shafer's upper and lower probabilities (16:2). The goal of the literature review is to examine the competing strategies in handling uncertainty in expert systems. A brief overview of how reasoning is accomplished is given and then the general theory of each methodology is outlined as well as its relative strengths and criticisms. Because this thesis will focus on a decision-analytic approach to expert system reasoning, this method is also reviewed and its suitability to the stated objective is given.

2.2 Reasoning Under Uncertainty

The presence of uncertainty in reasoning systems is caused by a variety of sources: the reliability of the information, the inherent imprecision of the representation language in which the information is conveyed, the incompleteness of the information, and the aggregation or summarization of information from multiple sources (29:854). How to represent this quality of belief is an ever-present problem in the reasoning process of expert systems; a problem that has yet to be solved in a universally accepted manner.

In logical systems, reasoning is performed in two manners—modus ponens and modus tollens. Considering two propositions p and q and assuming the implication $p \Longrightarrow q$ to be true, modus ponens allows the deduction of truth of q from the truth of p (see Figure 2.1). Conversely, modus tollens allows the deduction of falsity of

p from the falsity of q. These statements assume that the propositions are either true or false. This exactness is lost when dealing with uncertainty concerning the propositions. Consequently, given a proposition p and/or an implication p $\Longrightarrow$ q whose truth cannot be established definitively, (i.e., some other value besides 0 or 1) what can be said about q? (11:5)

Modus Ponens	Modus Tollens
$$\frac{\begin{array}{l} p \Longrightarrow q \\ p \end{array}}{q}$$	$$\frac{\begin{array}{l} p \Longrightarrow q \\ \neg q \end{array}}{\neg p}$$

Figure 2.1. Logic Reasoning

In situations like these, where knowledge is not definitive, uncertainty must be introduced in a manner that is consistent, consistent in the sense that the logic used propogates throughout the system in a coherent manner. The different schemes previously mentioned are methods which propose how expert systems should deal with this uncertainty.

2.3 Probability Theory

In his paper, "The Probability Approach to the Treatment of Uncertainty in Artificial Intelligence and Expert Systems," Lindley points to the inevitability and practicality of using probability in reference to uncertain events. With its sound theoretic basis, Lindley convincingly argues that probability theory is the only way to adequately discuss uncertainty (19:17-20). Henrion reiterates this position and notes that probability theory can be derived from a set of simple axioms governing rational decision-making under uncertainty. It is these axioms which form the basis of decision theory (12:2).

As it applies to reasoning in expert systems, probability offers definitive advantages over non-probabilistic measures of uncertainty. First, it provides a sound theory for using computed uncertainties in decision making. Second, it provides an operational definition for the likelihood (probability) of an event in terms of a person's behavior. Third, probabilistic schemes provide well-known ways of incorporating empirical data. And fourth, "there are well-developed methods for evaluating judged or computed probabilities by comparison with empirical frequencies, in terms of accuracy, resolution and calibration" (12:2). No non-probabilistic measures of uncertainty offer these advantages. Lindley, who generalized a scoring system to determine how good of an assessment different uncertainty methods are, points out that none do better than probability (19:27) (20:18).

2.3.1 Concepts of Probability The application of probability theory requires the prescription or the determination of some probability distribution over a set of objects (11:3). Each object is assigned a value between 0 and 1 such that for each object i the sum of the probability of i (P_i) would equal one ($\sum P_i = 1$). Bayes' theorem, which was developed by the Reverend Thomas Bayes, is the cornerstone upon which the foundation of probability reasoning rests. The theorem provides a method in which different probabilities can be combined in a consistent and coherent

manner.

According to a Bayesian, probability measures a person's degree of belief in a particular event or proposition, given the information currently known to that person. Mathematically, the theorem states that if A and B are two events, and that the P(A) is not zero, P(B|A) = P(A|B)*P(B)/P(A). (This is read as the probability of event B given event A is known.) Knue points out that there are only a few expert systems which strictly utilize sound Bayesian probability theory in their reasoning (10:3). This is surprising considering that Henrion claims that anyone who violates the law of probability and acts on incoherent probabilities, is liable to suffer demonstrable loss (7:2). Lindley's explanation of the "Dutch Book" demonstrates the necessity of having such a coherent theory (12:51).

2.3.2 Criticisms of Probability There are several criticisms against using Bayesian probability theory in expert systems. Below are the major ones as pointed out by Henrion (12), Goicoechea *et al.* (10), and Groothuizen (11):

(1) The rule requires all the hypotheses to be disjoint and, in a large expert system, dividing the solution space into mutually exclusive subsets may be expensive; (10:560)

(2) A large amount of statistical data is needed in the form of all $P(e_j|h_i)$; (11:3)

(3) In the event of altering the probability of an event in the system (by adding or removing hypotheses) all the probabilities would need to be recalculated; (10:560)

(4) There is no parameter that insures the set of probabilities built into an expert system is consistent and coherent; for example, the product P(A|B)P(B) may or may not be equal to P(B|A)P(A);(10:560)

(5) In realistic situations evidentiary information can quickly translate into very long sums and products of conditional and marginal distributions requiring substantial storage and computing resources;(10:560)

(6) Ignorance, vagueness, or second-order uncertainty cannot be represented; (12:4)

(7) In this approach: P(not $h_i \mid e_j$) = 1 - P($h_i \mid e_j$), that is evidence supports both h_i and (not h_i), although not to the same degree. This is intuitively very peculiar (i.e., this not how humans think); (11:3)

(8) It is difficult to combine evidence if it is not independent. Even then, it is not necessarily true that more evidence always gives more belief in a conclusion as probability theory tells us. (11:3)

2.4 Dempster-Shafer Theory of Evidence (Belief Theory)

The Dempster-Shafer theory of evidence (Dempster, 1967; Shafer 1976) is basically a set-theoretic generalization of Bayesian Theory (10:560). It involves establishing upper and lower probabilities which bound the subjective Bayesian probability for a particular hypothesis (16:23). Knue reports that this bounding gives some flexibility to inferences made in expert systems which use this technique (16:23). As noted by Gordon and Shortliffe (1984) the advantage of Dempster-Shafer theory over other approaches in handling uncertainty is its ability to model the narrowing of the hypothesis set with the accumulation of evidence (4:272). Shortliffe believes this process characterizes diagnostic reasoning in medicine and expert reasoning in general. Shafer's Belief theory has received considerable attention in recent years because it has a sound theoretical basis, it subsumes Bayesian theory, it is able to represent ignorance, and it allows beliefs from several sources to be treated symmetrically and pooled together (18:238).

2.4.1 Concepts of Dempster-Shafer. Dempster-Shafer theory is based upon a concept known as belief functions. A hypothesis is given a number in the range of [0,1] to indicate belief in the hypothesis given some evidence (4:275). "For every piece of evidence, a basic probability function (bpf) m: P(Θ) -> [0,1] is defined such

that m $[\Theta] = 0$ and $\sum$ m[A] = 1" (11:9). Θ represents the set of all hypotheses and $P(\Theta)$ is the power set of Θ which is summed over all subsets, A, of Θ. m[A] measures that portion of the total belief committed precisely to A. Unlike probability theory, m[A^c] $\neq$ 1 - m[A]. (The belief and the disbelief do not necessarily have to sum to one). The total belief in A is measured by the belief function Bel: Bel [A] = $\sum m[B]$ (11:9-10). After combining different evidences, a belief interval is defined which represents the strength of belief and disbelief in a hypothesis. See Groothuizen (11) or Knue (16) for further explanation on how evidence is combined and managed in expert systems.

2.4.2 Criticisms of Dempster-Shafer. There are several criticisms against Dempster-Shafer belief functions. Goicoechea *et al.* (10) and Groothuizen (11) point out some problems with the theory that need to be addressed in greater detail:

(1) Computation of combined bpf's is very time consuming; consequently in realistic cases a long chain of inferences may make the theory very inconvenient and expensive to use because of the increasing complexity in the structure of the core of the belief functions; (11:10)

(2) The definition of these combined bpf's indicates some kind of assumed independence; (11:10)

(3) Dempster's rule of combination cannot be applied in situations where there are considerable disagreements among the evidence, that is, when cores of two belief functions are disjoint (i.e., contradictory); (10:560)

(4) The numerical stability of the theory needs to be analyzed further; in some cases, small variations in the basic probability assignments can produce a large variation in the results. (10:560)

2.5 *Fuzzy Set Theory (Necessity and Possibility Theory)*

Fuzzy set theory, which was developed by Zadeh (1965), is another attempt to quantify uncertainty and imprecision in expert system reasoning. Fuzzy reasoning, which introduces the notions of necessity and possibility, looks at the world as not only being black and white, but as a continuum of grey (11:12).

Fuzzy sets differ from the classically defined set where a member can either be in the set or out of it, but not both. A fuzzy set is a class of objects which have various grades of membership in that set (16:17). The degree of membership is always a real number between 0 and 1, and it measures the extent to which an element is in a fuzzy set. In ordinary set-theoretic terms, the degree of membership measures the plausibility of an element being in a particular set (28:5). Consequently fuzzy sets are not mutually exclusive or exhaustive (16:17). If an element's membership is restricted to either 1 or 0, fuzzy sets reduce to normal sets. As a result, normal set theory is a special case of fuzzy set theory. A detailed analysis for defining fuzzy set membership functions can be found in Zimmerman (33) and Schmuker (28).

2.5.1 Concepts of Fuzzy Set Theory. Fuzzy set theory was developed in order to capture the vagueness inherent in some linguistic terms. This vagueness is evident when you try to quantify concepts like hot or cold, where hot in the context of weather is different than hot when used to describe a fusion reactor. Groothuizen (11) gives a good example using the word tall. The statement "John is tall" can be represented by: length (John) $\in$ A where length is known exactly and A is a fuzzy set, e.g.,

$$uA(x) = \begin{cases} 0 & \text{if } x < 170cm \\ 1 & \text{if } x > 190cm \\ or\,\frac{x-170}{20} & \text{otherwise} \end{cases}$$

Goicoechea *et al.* point out that there have been a number of applications of fuzzy logic to expert systems, including SP II and Reveal (10:560).

2.5.2 *Criticisms of Fuzzy Set Theory.*

Below are some criticisms against using fuzzy set theory in expert systems:

(1) The maximum and minimum rules for disjunction and conjunction may cancel valuable information when fuzzy individual assignments to various pieces of evidence include one assignment that is very close to zero; (10:560)

(2) Membership functions are context-sensitive; for example, a "small" building can be bigger than a "big" house; generic membership functions, if applied blindly, can lead to misleading results; (10:560)

(3) Computational and storage requirements can be large whenever individual membership functions are non-linear or non-trivial; discrete representation of functions is expensive; (10:560)

(4) Behavioral studies are lacking that would shed some light into the task of knowledge representation via membership functions by the user. (10:560)

2.6 *MYCIN Certainty Factors (Confirmation Theory)*

Certainty factors, which were developed to manage uncertainty in MYCIN, the medical consultant rule-based expert system, are the most popular of all the non-probabilistic techniques. Initially based on Bayes' Theorem, certainty factors were developed after the Bayesian method for updating evidence was considered inadequate. Buchanan and Shortliffe (4) give a thorough explanation of how and why certainty factors were decided upon and developed.

The basic principles behind certainty factors are the measurements of "belief" and "disbelief" of a particular hypothesis. Based upon Shortliffe's rules of combination, a number is reached which embodies the total belief in a particular hypothesis given all known evidence. Knue (16) does a good job of explaining the rules of combination for certainty factors as well as discussing some of its applications. Below is a quick overview of the system as presented by Goicoecea *et al.*

2.6.1 Concepts of Certainty Factors. MYCIN's theoretical framework includes terminology such as "measure of belief", denoted MB, "measure of disbelief", denoted MD, and "certainty factors", CF. Formally these are defined as: MB(H,E) is the measure of the belief in the hypothesis H, given evidence E; MD(H,E) is the measure of disbelief in the hypothesis H, given evidence E, and CF(H,E) = MB(H,E) - MD(H,E).

Since MB(H,E) is a number between 0 and 1, and MD(H,E) is also a number between 0 and 1, the certainty factor CF(H,E) is a number between -1 and +1. A positive CF indicates there is more reason to believe a hypothesis than disbelief it. A negative CF indicates that a hypothesis should be rejected more than confirmed. A CF of 0 is a "don't know" value which indicates that the hypothesis is independent of the evidence or equally supported (MB) and disconfirmed (MD)(10:561). Measures of belief are incorporated into the rules of the knowledge base and are used to propogate evidence towards a hypothesis. Though originally developed with medical diagnosis in mind, there are several non-medical expert systems which use certainty factors as the means of representing uncertainty.

2.6.2 Criticisms of Certainty Factors. The following are some criticisms against certainty factors:

(1) The definitions for combining the rules are rather arbitrary. Consequently, in some instances, results match an expert's intuitive feeling and in other cases, unexpected and unwanted results are yielded; (11:9)

(2) Combining rules assumes independence of the evidences; (11:9)

(3) It is does not have a strong theoretic basis; (10:562)

(4) The semantics of the CF, i.e., the interpretation of the number (ratio of probability, combination of utility values and probability);(29:857)

(5) Inability to distinguish between ignorance and conflict, both which are represented by a zero CF. (29:857)

This concludes the overview of some of the competing methodologies which profess to being able to represent uncertainty reasoning in expert systems. Because of the brevity devoted to each method, the referenced articles should be consulted for a more thorough understanding of particular concepts.

2.7 *Decision-Theoretic Approach (Decision Theory)*

As discussed by Hollenga and Morlan in their paper, "A Decision-Theoretic Model for Constructing Expert Systems," expert systems should use decision analysis as the methodology for dealing with uncertainty because of its sound theoretic base (13:2). Composed from the axioms of probability and utility, decision theory provides a framework for coherent assignment of beliefs with incomplete information (probability), while introducing a set of principles for consistency among preferences and decisions (utility).

2.7.1 Overview. The following paragraphs will summarize why a decision-analytic approach offers more than the previously listed alternatives for representing knowledge in expert systems. The strengths of probability and utility theory are briefly reviewed, and how the synergistic combination of the two in decision theory is both a viable method of reasoning and an accurate measure of uncertainty. Lastly the suitability of the approach for representing knowledge about the GPS satellite program is discussed.

2.7.1.1 ***Why Decision Theory uses Probability for Representating Uncertainty.*** There are a number of reasons why decision theory uses probability as its representation of uncertainty. Below is a list as presented by Horvitz *et al.* of fundamental properties that are considered intuitively desireable for measuring belief in the truth of a proposition.

1. Clarity: Propositions should be well defined.
2. Scalar continuity: A single real number is both necessary and sufficient for representing a degree of belief in a proposition.
3. Completeness: A degree of belief can be assigned to any well-defined proposition.
4. Context dependency: The belief assigned to a proposition can depend on the belief in other propositions.
5. Hypothetical conditioning: There exists some function that allows the belief in a conjunction or propositions, $B(X \wedge Y)$, to be calculated from the belief in one proposition and the belief in the other proposition given that the first proposition is true. That is, $B(X \wedge Y) = f[B(X|Y), B(Y)]$.
6. Complementarity: The belief in the negation of a proposition is a monotonically decreasing function of the belief in the proposition itself.
7. Consistency: There will be equal belief in propositions that are logically equivalent.

Researchers have demonstrated that, taken together, these properties logically imply that the measure of belief must satisfy the axioms of probability theory."... If one accepts these intuitive properties as desirable, one must then accept probabilities as a desirable measure of belief."(14:5) It is for these reasons Lindley (19) argues that probability is the only satisfactory description of uncertainty.

Using these properties as a gold standard, alternative methods for representing uncertainty can be judged in terms of which of the principles they reject. For example, Dempster-Shafer theory rejects the property of *completeness*. denying that it is possible to assign a belief to any well-defined proposition. Fuzzy-set theory rejects *clarity*, allowing linguistic imprecision in the definition of propositions. Certainty factors violate *consistency*, while other methods reject *scalar continuity*, arguing that a single number is insufficently rich to represent belief (14:5).

Concerning the critics of probabilistic methods in expert systems, who nonetheless want a numerical measure that reflects the state of a system's knowledge about, or attitude toward, a proposition's status, two problems arise. First, if they are not measuring probability, what are they measuring? It is not enough to associate a word ("confidence," "certainty," "degree of belief," "degree of confirmation," etc.) and hope that semantically it expresses the user's underlying intent. There is a well-developed theory of statistics, with interpretations for the precise meaning of "probability" as well as proven calculi which govern its use. Any competing theory will have to present the same thing: not merely a name, but a clear analysis of what is being measured, how it is different from a probability, why people should nonetheless be interested in it, and how people can feel certain that computations used to derive the numbers in fact compute anything worth value. To date, most of the work in these directions remains suggestive rather than persuasive (29:846).

2.7.1.2 Utility Theory Provides a Rational Framework for Reasoning. Utility theory is based on a set of simple axioms or rules concerning choices under uncertainty. Just like the axioms of probability theory, they are fairly intuitive. These axioms provide a model of how a rational man makes his decisions when faced with uncertainty. For the purpose of decision theory, the most important thing to note is that the axioms of coherence imply that a person should make decisions in such a manner as to maximize the expected utility (EU). If one accepts the axioms of coherence, he must accept the EU criterion, because it follows logically from the axioms.

The following is a brief summarization of the axioms of coherence, as presented by Winkler (32). See Winkler for further details or clarification.

Axiom 1: Given any two payoffs R1 and R2, you can decide whether you prefer R1 to R2, R2 to R1, or you can be indifferent between R1 and R2.(orderability)

Axiom 2: If you prefer payoff R1 to R2 and you prefer R2 to R3, then you must prefer R1 to R3 (transivity of preferences). This is a very important element in the theory of subjective probability and utility.

Axiom 3: If you prefer R1 to R2 and R2 to R3, then you can find some probability value p, such that a p-mixture of R1 and R3 is preferred to R2; you can find some other value of p such that R2 is preferred to a p-mixture of R1 and R3; and finally you can find yet another value of p such that you are indifferent between R2 and a p-mixture of R1 and R3.

Axiom 4: If R1 is preferred to R2 and R3 is some other payoff value, then any p-mixture of R1 and R3 is preferred to the same p-mixture of R2 and R3 (where p value is the same).

Axiom 5: If you are indifferent between R1 and R2, then they may be substitued for each other as payoffs in any decision making problem.

Axiom 6: If R1 is preferred to R2, then a p-mixture of R1 and R2 is preferred to a q-mixture of R1 and R2 if and only if $p > q$.

The six axioms of coherence appear intuitively reasonable and as such should be adhered to in rational decision making. It seems counterintuitive and illogical to develop an expert system which does not reason coherently.

2.7.1.3 Sound Recommendations Under Uncertainty. Decision theory does not claim to provide a description of how people actually behave when reasoning under uncertainty, but how they should. Horvitz *et al.* refer to studies which have demonstrated that people frequently do not behave in accordance with decision theory. In fact, characteristic (and often costly) biases exhibited in intuitive judgement are part of the justification for applying decision theory to assist people in making decisions. While much can be done to determine the probabilities, structure the values, and assess the alternatives, bad outcomes still can result from rational choices. Alternatively, a random or poor selection, may turn out to be quite

fortuitous. Such is the nature of acting under incomplete information. Nonetheless, decision theory strives for good decisions that lead to better outcomes on average (14:9).

2.7.2 Suitability of GPS. Though opponents argue the inadequacy of the decision-analytic method because they feel it requires too much prior information (a criticism levied at all probability-based systems), it appears that the GPS satellite program provides a suitable environment in which this information can be ascertained. As for disagreements with the chosen methodology, Henrion (12) effectively argues that several of the criticisms levied against probabilistic reasoning in expert systems are unwarranted and untrue. In particular, he references the work of Cheeseman (1985) and Spiegelhalter (1986) who show that ignorance, vagueness, and second-order uncertainty can be represented in a sound probabilistic manner (12:5). Pearl's (24) Bayesian networks demonstrate that probabilistic representations of information do not require the massive amounts of data that many claim. Snow's (30) work reiterates this point. As for criticisms concerning the intractability of the theory, Kim and Pearl's method for propagating uncertain evidence through a Bayes' network show that it is not necessary to recalculate all probabilities as new information is acquired—only those probabilities directly related to the new information (12:3).

Concerning complaints associated with knowledge acquisition, it appears as if the GPS satellite's extensive historical database make computing prior probabilities a statistical problem while utility assessments can be provided by expert opinion. As for its applicability to expert system reasoning, Breese and Fehling point out that in situations where the cost of an error is important, probabilistic reasoning founded upon decision theory is the preferred method of uncertainty reasoning (3:33). Kalagananam and Henrion (15), using motorcycle repair as the model, have demonstrated the advantages of using a decision-analytic approach versus a heuristic approach for sequential diagnosis. Edmond's (8) study concerning satellite anomaly resolution

also confirms this point, while Spiegelhalter (31) has also shown the utility of this approach in medical diagnosis.

As for the difficulty of generating rules which capture this methodology, Hollenga's (13) proposed method of developing rules from a decision analytic action diagram appears promising and warrants further study.

Overall, the decision-theoretic approach, unlike many alternative formalisms of uncertainty, provides a coherent prescription for choosing actions and meaningful guarantees of the quality of these choices (29:55).

2.8 Summary

This literature review has given some background on what the principle methodologies are for dealing with uncertainty in expert systems. Though this list is by no means exhaustive, Bayesian probability, Demster-Shafter belief functions, Certainty Factors, and Fuzzy Set Theory represent a few of the more prominent theories. Although successful in limited applications, each method still has criticism as to its overall effectiveness in varied fields.

In the section concerning the decision theoretic approach to handling uncertainty, several points were made that confirmed decision analysis is a valid approach to uncertainty reasoning in expert systems. For one, the decisions made by this probabilistic method can be communicated in intuitively meaningful terms. Two, the assumptions leading to these decisions can be traced back with ease and clarity. And three, the ability to assess the utility of certain decisions, represents value judgements about the preference and desirability of actions in the context of their perceived worth (29:56). Though not above reproach, decision theory can nonetheless rectify the shortcomings of other uncertainty methods by providing a set of well founded principles (3:36). The next chapter will discuss the methodology used in this thesis.

III. Methodology

3.1 Introduction

This chapter outlines the methodology used in the thesis. Decision theory is the framework of the research with particular emphasis on subjective Bayesian Reasoning. If unfamiliar with the major concepts of decision theory or Bayesian reasoning, the following references give adequate presentations of major concepts these theories entail. See Horvitz *et al.*(14), Sharpiro (29), and Winkler (32).

The following paragraphs address the approach needed for satisfying the sub-objectives of the thesis. It is the framework of these sub-objectives that form the scope of this thesis. Consequently, in order to determine the feasibility of using a decision-analytic approach in developing rule-based expert systems, each area will be examined. The addressed areas are as follows:

1. The suitability of the decision-analytic approach;
2. The present methodology behind GPS anomaly resolution;
3. The formulation of the needed probabilities from the historical satellite data; and
4. The generation of rules which capture GPS knowledge for preliminary expert system prototype. A discussion and summary of the results is also necessary.

3.2 Applicability of the Approach

Because Bayesian theory is based upon fundamental coherent mathematical concepts, this thesis will determine if it offers a sound basis upon which an expert system can be developed. This will involve examining how the various probability and conditional relationships are developed and how they interact. This will also involve determing if the system knowledge can be adequately expressed in a probabilistic manner.

Certain assumptions are needed in order for Bayesian reasoning to be useful, therefore it will be determined if this particular type of problem meets those criteria and what are the system limitations.

3.3 Anomaly Resolution

Presently GPS anomaly resolution is accomplished in two phases—each with several steps. Upon noticing anomalous telemetry, phase one is for the operator (usually not an expert) to infer what class of problem he potentially faces based upon the anomalous telemetry. This leads to a checklist which, written as a flowchart, guides the operator to an action which will either remedy the problem or put the vehicle in a safe configuration. If it is necessary to "safe" the vehicle, phase two is for an expert to review the evidence (telemetry) prior to the anomaly, decide what went wrong, and based upon the subjective utility of a particular action, decide what should be done. This is a heuristic process which relies upon the expert's ability to reason with incomplete or uncertain data and reach a decision which maximizes the utility for the satellite and the user. For this research, interviews with GPS subsystem engineers were conducted in order to understand what processes are involved in making decisions which govern the well- being of the satellite. This helped define the reasoning process that the expert system should follow and also in ascertaining the utility value of selected actions. A review of the vehicle subsystem technical manuals as well as contingency procedures also provided needed background knowledge.

3.4 Data Generation

Based upon archived data of a satellite's history, the actual prior probabilities needed in Bayesian analysis were computed. Expert opinion as well as past trends allowed accurate probabilities of particular anomalies and their interrelationships to be determined. This facet of the research was crucial to the overall success of the project. Interviews with GPS vehicle engineers as well as a review of statistical data

collection methods were required in order to validate results.

3.5 Rule Base Development

A small decision-analytic reasoner, which combines information in a Bayesian manner, was developed and a basic example using data obtained from GPS engineers was coded. This facilitated testing and validating the approach. It also helped in determining the utility of this approach in representing system knowledge and handling uncertainty in expert systems of this type.

3.6 Summary

Throughout this thesis project, an effort was made to bridge the gap between decision theory and expert system development. With satellite fault diagnosis as the chosen domain of application, this thesis hopes to show that these areas can be merged in a practical and useful manner.

IV. Global Positioning System (GPS)

4.1 Introduction

The purpose of this thesis is to determine if a decision theory approach to rule-based expert systems is feasible and practical. The Global Positioning System (GPS) satellite is the system chosen to determine the viability of the approach. Anomaly classification and present contingency procedures describing GPS satellite operations are reviewed in order to determine the scope needed by the expert system. Background knowledge on GPS and its mission, why this satellite program was singled out, and which subsystem will be examined for rule base development is also given.

4.2 Anomaly Classifications

As defined in Section 3.0.2 of the GPS Orbital Operations Handbook (OOH) (27), an anomaly is the improper or unexpected operation, condition, or behavior of any portion, subsystem, or system of a space vehicle. An anomaly may manifest itself in an out-of-limits parameter or adverse trending of any parameter, as indicated in telemetry data; failure of the space vehicle to respond to commanding; off-nominal attitude or orbit conditions; or non-nominal indications obtained from the GPS Control Segment or Camp Parks Communications Annex. The various levels of anomalies are defined as:

1. ***Level I Anomaly.*** A Level I anomaly could cause a total loss of mission and requires an immediate response.
2. *Level II Anomaly.* A Level II anomaly could cause a total loss of mission but allows some time to respond.
3. *Level III Anomaly.* A Level III anomaly could cause a problem less severe than a total loss of mission and requires an immediate to long-term response.

4.3 Discussion

The problems considered in this thesis are Level II and III anomalies. The assessment of utility values pertaining to contingency actions concerning the anomaly, can be weighted in order to examine problems of a higher priority before examining those of lower priority should instances arise where evidence lends itself to different problems. However, in the Orbital Operations Handbook (OOH), there is no explicit utility scale ranking for anomalies except for the Level I, Level II, and Level III classifications. Within each of these levels there are none. This knowledge is something that is supposed to be learned as the operator becomes more familiar with the outcome of certain problems. Likewise, recognizing the boundary between the different classes starts to blur as time is factored into the problem. Consequently a Level III problem undetected can manifest itself as a Level II anomaly and so forth. The heuristic experience that comes from long term program association is the type needed to be captured in a expert system knowledge base.

Presently, anomalies are resolved using contingency checklists which have enumerated most of the foreseeable problems. These checklists are written in a posterior type manner—hypotheses are concluded based on the evidence observed by the operator. They are rigidly written and appear to allow for no subjective interpretation. However, this aspect is aided by human judgement as the operator gets more familiar with the telemetry readings. For example, if the checklist requires that telemetry point $X = 0$, and the user knows that 0.1 is essentially zero due to the particular traits of that vehicle, he considers this value as zero and proceeds. This type of subjective assessment is what is lacking in many expert systems which, like the OOH checklists, are written in a rigid form and fail to allow for the belief values to be coherently incorporated into the knowledge base. As mentioned previously, the probabilistic representations of uncertainty by decision theory models can be used to encompass this type of data assessment.

Presently, anomaly resolution in all but the simplest of cases usually requires a group decision before major corrective actions are initiated. Due to the nature of most problems and the inherent reliability of the satellite, usually no important decisions have to be made in such a short time that the situation cannot be throughly examined before proceeding. In situations like these, an expert system can be used to augment the knowledge of the operations crew. Nonetheless, there are problems which do require immediate action, and the operations crew must be proficient in performing the necessary tasks. For situations like these, consultation with an expert system may be inappropriate due to the criticality of time.

4.4 What is GPS

The Navstar GPS is a Air Force operated satellite program whose mission is to provide highly precise position, velocity, and time information to users around the world and to detect nuclear detonations (NUDETs). The GPS system is made up of four system segments: Space Segment, Control Segment, Navigation User Segment, and NDS User Segment.

4.4.1 Space Segment. The Space Segment, when fully operational, will consist of 21 Block II operational satellites and three active spares placed in six orbital planes, each having an inclination of 55 degrees. The satellites will operate in circular 20,200 kilometers (10,900 nautical miles) orbits with a period of 12 hours (6:2). The satellites will be spaced in such a manner that a minimum of four satellites will be visible by any user at any time, thereby ensuring worldwide coverage.

The GPS operates on two L-band frequencies: 1575.42 MHz (L1) and 1227.6 MHz (L2). Each satellite is designed to transmit an L1 and L2 signal. L1 carries a precision (P code) signal and a coarse/acquisition (C/A code) signal, while L2 carries the P code only. The use of two frequencies allows the navigation user with a higher accuracy set to adjust to the variable delay experienced by different signals as they

pass through the ionosphere. Superimposed on these signals will be navigation data, the satellite's current ephemeris coordinates (position), state of health information, and satellite clock bias information.

In the NUDET detection role, each satellite utilizes onboard sensors to detect nuclear events on or above the earth's surface. A satellite detecting the nuclear events processes and crosslinks the data to other GPS satellites in radio frequency (RF) proximity via UHF frequencies. All satellites with the nuclear event data transmit to the NDS User Segment via the L3 frequency.

4.4.2 Control Segment. The Control Segment includes a number of monitor stations and ground antennas located throughout the world. The monitor stations use a GPS receiver to passively track all satellites in view and thus accumulate ranging data from the satellite signals. The data is also processed to determine each satellite's precise ephemeris coordinates and the prevailing time registered by the onboard atomic clock. Detected errors are corrected by transmitting new upload data to the satellite on a periodic basis. The state-of-health of the satellite subsystem and payloads are also monitored.

4.4.3 Navigation User Segment. The Navigation User Segment receives and processes L1 and L2-band navigation data radiated from the Space Segment. The user passively receives coded signals from four or more satellites in view of his receiver. The coded signals from three satellites are used to triangulate the user's velocity and coordinates. Data from the fourth satellite is used to eliminate any timing errors in the user's set clock.

4.4.4 NDS User Segment. In support of the Nuclear Detonation (NUDET) Detection System (NDS) mission, the GPS Block II satellite provides the capability to transmit Global Burst Detector (GBD) collected NUDET data to NDS User Segment terminals by direct transmission using the L3 downlink and through trans-

mission by GPS satellites within RF proximity via the UHF crosslink. NDS users process these data together along with navigation data to determine the location and characteristics of the nuclear event(s).

4.5 Applications

Because of the versatility and the increased navigational capability it offers, GPS provides many benefits to both civilian and military operators. Knowledge of precise three-dimensional position data relative to friendly and enemy forces is fundamental to the success of a large number of military missions, while increased commercial traffic in the air and on the sea can utilize this information for increased safety and efficiency.

4.5.1 Military Applications. The substantial navigation performance improvements afforded by the GPS can enhance many areas of military operations. In air operations, GPS accuracy can streamline en route and terminal navigation, thereby reducing flight times and conserving fuel. Because GPS is a three-dimensional system, descent and landing operations can be more closely controlled. Combat-related applications involving bombing and ballistic weapon delivery are also vastly improved. GPS also allow ground forces the capability of enhancing activities such as site surveying, field artillery placement, and target acquisition and location. Naval forces also benefit. Harbor entry operations can be improved and activities such as mine placement can be made more precise and safe. These are but a few of the many military applications that will benefit from GPS (see Figure 4.1) (6:9).

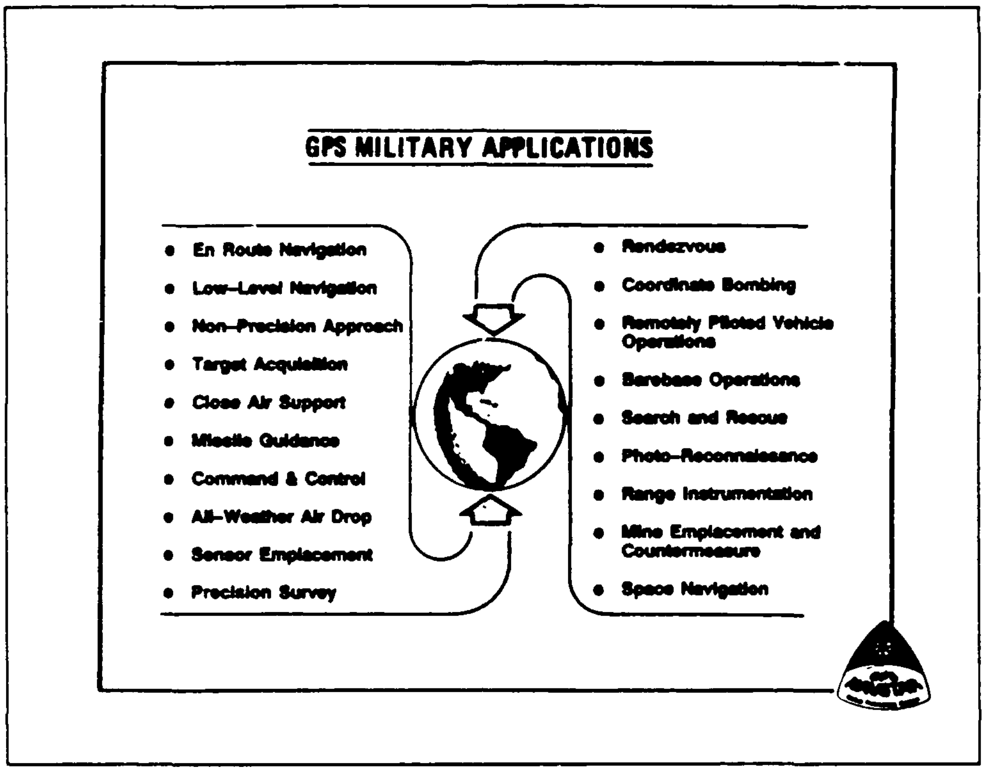

Figure 4.1. Military Applications

4.5.2 Civilian Applications. The GPS will provide a broad spectrum of civil users with accurate position, velocity, and time determination cabability at a reasonable cost. Civil users of air, sea, and ground vehicles will benefit from the GPS for optimal course navigation, which will reduce fuel costs and transportation time. Besides providing substantial benefits in air navigation and landing operations, air search and rescue techniques will also be greatly improved. Mineral exploration, and accurate positioning of oil exploration are also some other commercial uses of GPS. There are several potential applications of GPS that will occur as the system becomes more accepted by the civilian community (see Figure 4.2).

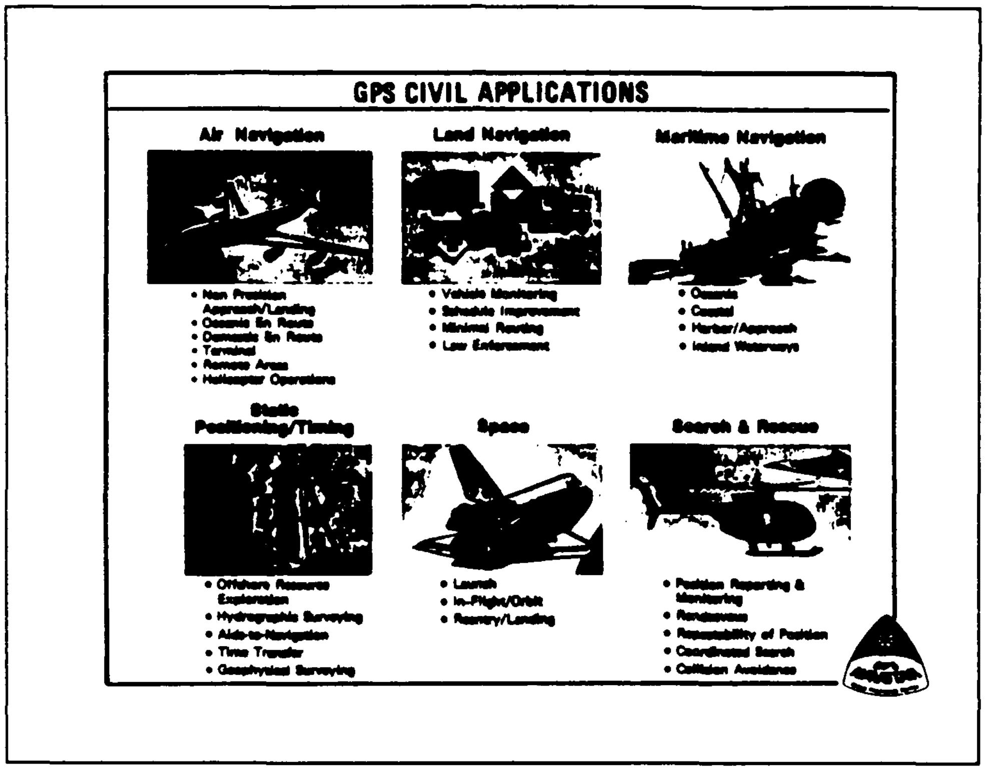

Figure 4.2. Civilian Applications

4.6 *Space Vehicle Subsystems*

GPS Block II satellites consist of a integrated assembly of nine subsystems and two payloads to provide a space-based radio frequency enviroment for the Global Positioning System.

The space vehicle consists of the following subsystems and payloads:

1. Structure Subsystem (STR)
2. Thermal Control Subsystem (TCS)

3. Electrical Power Subsystem (EPS)
4. Attitude and Velocity Control Subsystem (AVCS)
5. Orbital Insertion Subsystem (OIS)
6. Reaction Control Subsystem (RCS)
7. Telemetry, Tracking, and Command Subsystem (TT&C)
8. Navigation Payload (NPD)
9. L-Band Subsystem (LBS)
10. Integrated Transfer Subsystem (ITS)
11. Global Burst Detector (GBD)

The AVCS, OIS, and RCS subsystems are elements of the control system (CS). The ITS and GBD subsystem are part of the Nuclear Detonation (NUDET) Detection System (NDS) payload (27). See Appendix A for a summary of each subsystem as given in the GPS Orbital Operations Handbook (OOH) Section 1.0.

4.7 Why GPS?

The Global Positioning System was chosen as the prototype model for application of the decision-analytic approach to rule-base development for a number of reasons. The main reason is that the Air Force is becoming more reliant upon space-based resources and expert systems will play an increasing role in managing and maintaining these systems. Without question, GPS is one space resource which figures to play a very important role in future military operations. Consequently, the viability of this approach in augmenting decision-making and system control needs to be examined in the context of its possible uses. Another strong point is that because the first generation of GPS satellites (Block I) have been in an operational test phase since 1978, there is a large data base from which system anomalies and trends can be determined. This provides the information needed to ascertain the utility of certain actions as well as form the priors and likelihood ratios needed. And lastly, because a satellite is a complex system, the feasibility of the approach on large systems can be determined.

4.8 System Examined

For this study, the Electrical Power Subsystem concerning the loadshed timers will be the chosen area in which contingency system knowledge will be modeled. After conferring with Technical Analysts from Lockheed Missile and Space Corp., the EPS system was chosen because of its importance, and because most of the common anomalies occur in the EPS. Furthermore, extensive data concerning the EPS subsystem have been kept which will assist in determining the needed conditional probabilities of anomalous conditions given telemetry.

4.9 Conclusion

The purpose of this section of the thesis was to give background information on the GPS satellite system and the reasons it was selected as the model. Anomaly classifications and contingency procedures were reviewed as well as the applicability of satellite expertise to expert system technology. The Electrical Power Subsystem was chosen as the subsystem in which a knowledge base will be developed to test the viability of the method. The next chapter discusses the assumptions that the decision-analytic model is be based upon.

V. Model Description

5.1 Introduction

This section of the thesis discusses the decision-analytic approach to developing expert system rules which capture the knowledge of a particular problem domain. The framework is that of decision analysis encompassing Bayesian reasoning and utility theory. The strengths of the methodology are that uncertainty can be incorporated into the decision-making process in a consistent manner; the outputs can be validated and verified; and, given the initial model accurately describes the problem domain, the method is fairly robust. This chapter addresses the assumptions upon which this system is based, why a system with its capabilities are needed, and the major differences between this rule base and more traditional ones. Morlan's (22) in-progress work was used as a guideline for the layout of the initial part of the chapter.

5.2 Overview

Bayes' theorem is the foundation upon which this approach to rule-based expert system development is based. Should a review of this theorem and its basic concepts (i.e., priors, likelihoods, posteriors) be required, Berger (1) or Morgan (21) are recommended references.

Bayesian analysis represents a method for incorporating available relevant information directly into the process of making inferences about an underlying state of nature, and in formulating decisions based on these inferences. Furthermore, it allows for taking account of subjective belief in the evidence (second-order uncertainty) in arriving at these inferences. As mentioned in the literature review, several methods of incorporating this uncertainty have been developed, yet few if any do so in a coherent manner and fewer still use utility as a measure of delineating between

choices. This approach of managing knowledge in expert systems attempts to operate in a manner that more closely resembles rational decision-making. In so doing, the knowledge base, which has probabilistically captured domain-specific knowledge, coherently manipulates a user's belief about the evidence and recommends the action yielding the most utility. However, unlike conventional expert systems, these recommendations are made from rules which are more analogous to mathematical functions than "If ... then..." statements. Whether this rational approach toward uncertainty management can be sucessfully mapped into human reasoning, which is many times an illogical and incoherent process, remains to be seen.

5.3 Assumptions

These are the assumptions that are used in the application of this method:

1. Hypotheses are assumed to be disjoint when the conditional probability and conditional utility information is gathered,

2. Second-order uncertainty (uncertainty in the evidence state) is independent of the hypothesis-to-evidence uncertainty,

3. Actions are assumed to be disjoint,

4. Evidences are not necessarily conditionally independent within a hypothesis (i.e., P(E1,E2|H) might or might not equal P(E1|H)*P(E2|H),

5. Only a relevant subset of the hypotheses needs to be considered.

*5.3.1 **Disjoint Hypotheses.*** Assuming disjoint hypotheses is a fairly common way to decompose problems. It allows for subjective reasoning about uncertainty by collecting the conditional probability (P[E|H]) of the evidences, E, conditioned on the hypotheses, H (the likelihoods). That concept is also extended to the utility information by the conditional utility function, U[A|H].

Some of the strong points of this assumption are:

- It provides a natural decomposition of larger problems into smaller more manageable ones;
- It is a logical partition for collecting evidence-to-cause probability relationships;
- It is a logical partition for collecting action-to-cause utility information.

Some of the disadvantages are:

- The requirement to construct a separate hypothesis that represents the joint occurance of hypotheses that may occur together;
- The requirement to include data on all possible hypotheses. However, this task is required of any type of expert system that strives to make rational decisions.

5.3.2 Second-Order Uncertainty. Second-order uncertainty is uncertainty concerning the state of the evidence. By using an alternate form of Bayes' Rule, sometimes noted as Jeffrey's Rule, uncertainty can be applied to the evidence in a consistent manner that adheres to the laws of probability yet allows for subjective opinion to be addressed. Mathematically, Jeffrey's Rule is (7:2)

$$P\star(A) = \sum_{i=1}^{n} P[A \mid E_i]\, P\star(E_i).$$

Though apparently sound in its adherence to probability theory, there is still some disagreement among probablisticians as to its validity or need. Pearl (24) and Snow (30) have professed differences to Jeffrey's Rule and Morlan (22) is presently working on a concept which contradicts with Jeffrey's belief in the relationship of different evidences. Nonetheless, this thesis used Jeffrey's Rule to demonstrate the concept of modifying the belief in a hypothesis given the uncertainty in the evidence. See referenced articles for alternate methods of incorporating additional information.

The following examples demonstrate a objective method toward handling second-order uncertainty by utilizing Jeffrey's Rule. Example 1: Assume P[H1| E2] = .7, P[H1| -E2] = .262. Now, suppose your belief in the evidence (E2) is sixty percent (i.e.,"P[E2]" = .6). How can this belief be coherently incorporated in the posterior of the hypothesis given the evidence? Bayes' (Jeffrey's) Rule allows us a manner in which we can reassess this belief:

"P[E2]" = .6 therefore "P[-E2]" = .4

$$\begin{aligned} P[H1 \mid \text{“}P[E2]\text{”}] &= P[H1 \mid E2] \cdot \text{“}P[E2]\text{”} + P[H1 \mid -E2] \cdot \text{“}P[-E2]\text{”} \\ &= (.7)(.6) + (.262)(.4) \\ &= .42 + .1048 \\ &= .5248 \end{aligned}$$

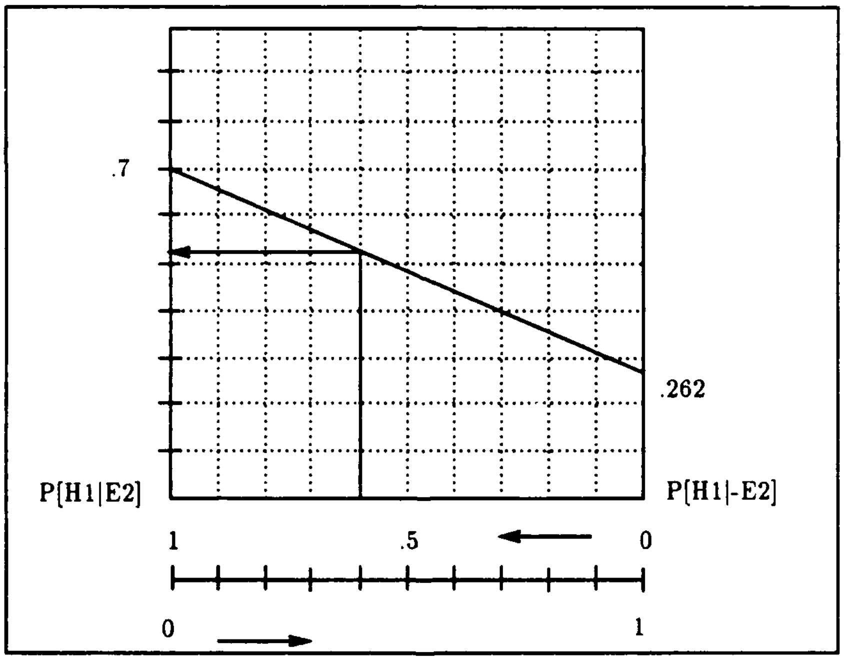

Figure 5.1. Jeffrey's Rule 2D Representation

Because of the uncertainty of the evidence, the belief in the hypothesis should fall somewhere between the extremes of when you are sure of seeing the evidence

P[H1| E2] and when you are sure you did not see the evidence P[H1| -E2]. See Figure 5.1 for graphical interpretation.

Example 2: Assume

$$
\begin{aligned}
P[H1 \mid E1, E2] &= .9 \\
P[H1 \mid E1, -E2] &= .8 \\
P[H1 \mid -E1, E2] &= .7 \\
P[H1 \mid -E1, -E2] &= .6
\end{aligned}
$$

Suppose: "$P[E1]$" = .3 and "$P[E2]$" = .2
Therefore: "$P[-E1]$" = .7 and "$P[-E2]$" = .8

Then: $P[H1 \mid \text{"}P[E1]\text{"}, \text{"}P[E2]\text{"}] =$

$$
\begin{aligned}
&= (.9)(.3)(.2) + (.8)(.3)(.8) + (.7)(.7)(.2) + (.6)(.7)(.8) \\
&= .054 + .192 + .098 + .336 \\
&= .68
\end{aligned}
$$

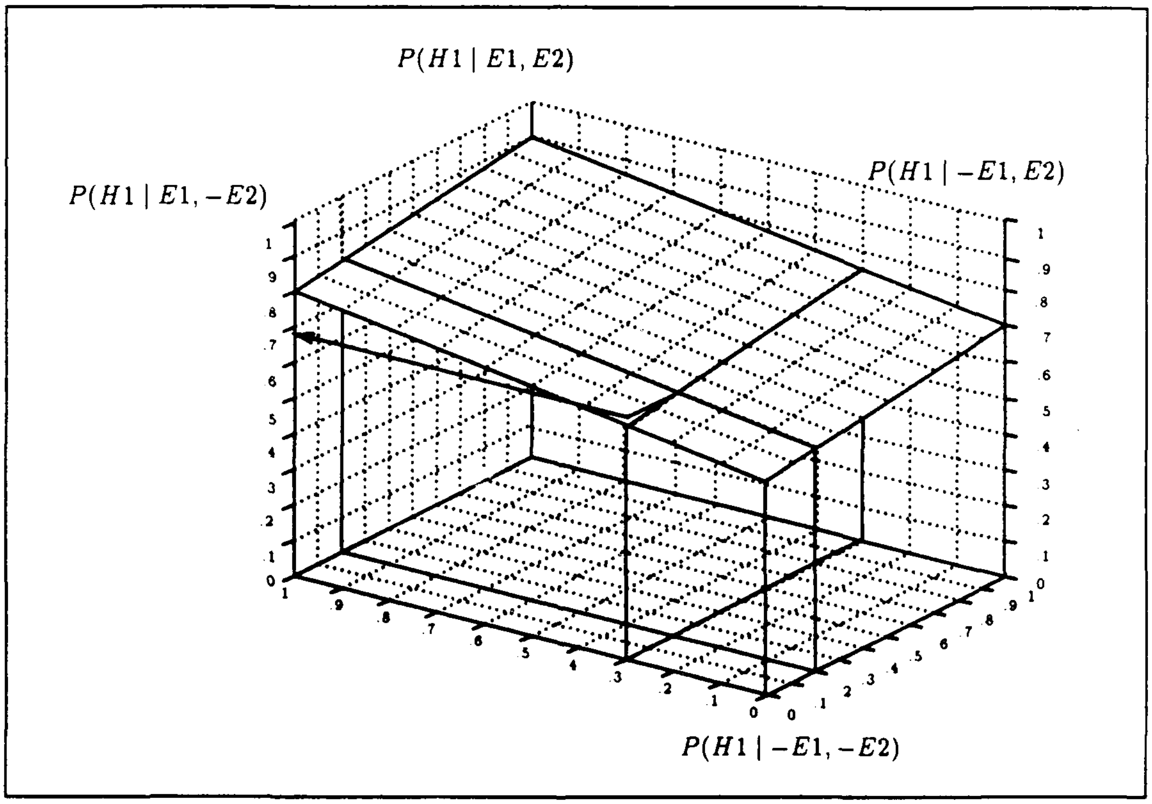

Figure 5.2. Jeffrey's Rule 3D Representation

This value is between P[H1|E1,E2] = .9 and P[H1|-E1,-E2] = .6 just as would be expected. Figure 5.2 shows the three dimensional representation. This manipulation of uncertainty will be discussed later in the actual problem application.

5.3.3 Disjoint Actions. The assumption that the actions are disjoint is similar to the assumption that the hypotheses are disjoint in that, without it, the solution space would be combinatorially large. Given N actions, a data base allowing all combinations would be 2^n. For purposes of this thesis, only one action is recommended. It is understood that this might be unrealistic in terms of real world situations.

5.3.4 Conditional Independence. Most Bayesian problems assume evidences are conditionally independent. In such a case, the joint probabilities are simply the individual probabilities multiplied together (i.e., P[E1,E2 |H1] = P[E1|H1]*P[E2 |H1]). Many times this is indeed a valid assumption which helps in combining several pieces of information. Snow (30:635) points out several ways in which data can be manipulated into satisfying this simplifying assumption. For one, the evidences may be restated or algebraically transformed so that they are conditionally independent or it may be possible to combine dependent evidences into one evidence which can be considered independent across the hypothesis set. All in all, "Performing statistical inference under the conditional independence restriction is a lot like modeling physical systems using linear relationships. In both cases, the restrictive assumption makes the output a simple combination of the responses for each separate input." (30:635) Nevertheless, in many real world situations there are times when this assumption cannot be made. When situations are encountered where evidences are dependent given an hypothesis, their relationships must be accurately captured in the knowledge base if the expert system is going to provide realistic assistance. The model does allow for the accurate representation and manipulation of dependent data.

5.3.5 Plausible Set. Given a certain problem domain (subset of large system), heuristics are used to narrow down a set of expected hypotheses which pertain to the domain in question. If this assumption is not made, the database would become too large trying to enumerate every possible (no matter how unlikely) state of nature (hypothesis).

This "pruning" of the hypothesis space, which is a subjective process in itself, would involve determining which hypotheses are deemed most likely to occur. As in traditional rule-based systems, this "plausible" set of hypotheses is based upon past history, similar problems, or educated guesses. However, unlike traditional rule-based systems, the utility of not considering certain hypotheses must also be factored into the decision of determining the plausible hypothesis set. Some threshold level must be decided upon that takes into consideration the likelihood of the hypothesis and also the expected loss should that hypothesis not be considered. By examining utility information, hypotheses which might be excluded because of their remoteness might enter into the plausible set because they are just too "expensive" to not account for their happening.

5.4 Justification

As mentioned previously in the literature review, there are several reasons why decision theory is a desirable approach in expert system development. The primary reason is the ability to coherently handle uncertainty. The following paragraphs examine the fundamental differences between this type of system and more traditional ones, and why this approach appears more suitable to real world applications.

As with any knowledge base that attempts to model complex systems, a definitive approach must be used in the formulation of rules. Generally, there are two ways, one which does not incorporate uncertainty or implication and one which does; the former being superficial yet functional in ideal situations, the latter being a great deal more complex yet incorporating a more realistic attitude toward quantifying

knowledge.

The first approach merely looks at indicators which are believed to be symptoms of suspected problems. Much like the belief that a fever is a symptom of some type of condition (e.g., measles, flu, etc.). This approach accepts the indication (evidence) of the fever as being true and then proceeds to determine what would cause it and subsequently how to treat the condition. A rule might be "If patient has a fever of > 99.3, then prescribe aspirin." No allowance is given for subjective assessment considering external situations which might effect the strength of the implication or the user's belief. Most rule-based systems are one dimensional in this respect and operate accordingly, many, it must be noted, with quite satisfactory results.

The other approach is more complex, looking at each piece of evidence in the context of the entire situation. Given the same situation of a person having a fever, this approach would allow subjective questioning of the evidence (i.e., how accurate is the thermometer? or who took the reading; was it the doctor (expert), or a new candy striper?). Given a rule base which is formulated in this manner, each piece of evidence is able to be judged as to its veracity and this interpretation can be used to determine the fault (anomaly) and subsequent action. Consequently, while the implied relationship of 99.3 degrees and fever is stored in the data base, your belief (uncertainty) might waiver anywhere from 1.0 (definite fever) to .2 (most likely not a fever) to 0.0 definitely not a fever, depending on your assessment of the situation. This degree of belief could cause your next action to range from prescribing aspirin to sending the patient home. The advantages of a system which can reason in such a manner are many, however there are disadvantages; one being that the inference engine needed is quite complex and two, the size of the rule base is quite large. Nevertheless this is to be expected from any system which tries to approach a human level of reasoning capability.

This decision-analytic model, unlike alternative rule-based methods for knowledge representation and handling uncertainty, does allow for the incorporatior. of

other information in a manner which adheres with the expert's opinion yet consistently propagates this belief throughout the system. Furthermore, the strength of the implication within individual rules can be controlled by the likelihood data.

5.5 *How the Rule Base Differs*

The rules which operate in this expert system environment are not of the context "if A then B". They function in the same manner, yet are not as simple in structure nor form. Instead they operate more along the lines of mathematical functions than simple logical statements. This allows the rule base to be more dynamic and flexible in its manipulation of data.

Stored probabilistically in the system is the basic framework from which an expert has determined which situations warrant what actions, and what evidence can be used to determine the needed action. This knowledge is expressed in the prior, likelihood, and utility data. Using a decision theory approach, recommended actions are based on the highest utility. These recommendations can vary according to the $\sum P(H \mid E) * (Ua)$. Consequently, varying the probabilities concerning the evidence about the state of the world may lead to different actions. As seen in Figure 5.3 below, various values of E_1 and E_2 would cause different courses of action to be pursued. Because of this functional relationship, the rules are not all enumerated beforehand, but merely the expert's functional belief as to how the information sh uld relate together.

Comparing this output with traditional rule bases, it can be shown that these rules provide all the capability of those written in the rigid "If A then B" format, yet offer much more latitude in fault diagnosis because second-order uncertainty can be incorporated. Needless to say, this is precisely the type of system needed in an area such as satellite operations, where human judgment must temper the information seen in the context of the environment.

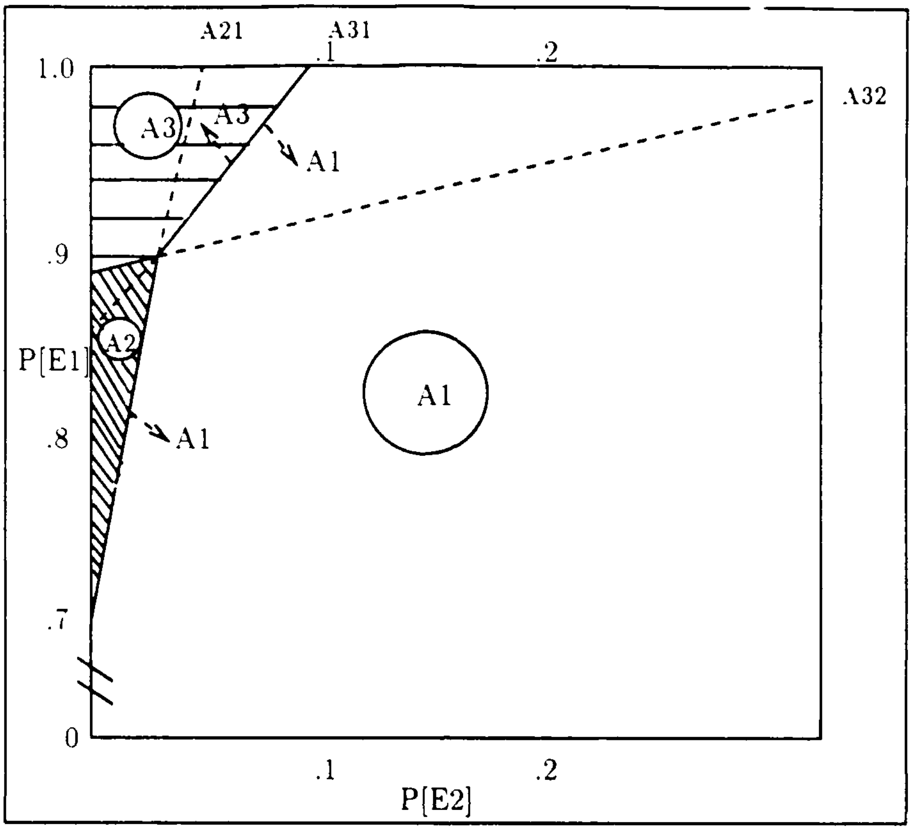

Figure 5.3. Action Diagram

5.6 *Knowledge Base*

The knowledge base is made up of three parts: the prior distribution of the plausible hypothesis set, the joint likelihood matrix, and the utility table values. Because the joint likelihood matrix represents the bulk of the information of the knowledge base, and yet differs largely in comparison with alternative rule-based data bases, its make up and use will be discussed. Particularly how various combinations of the data are represented given both independent and dependent evidences.

Given the individual likelihood values provided by the expert, three joint matrices are created representing different versions of likelihood information (see Figure 5.4). These are denoted as the P(E|H), the P(-E|H), and P({E}|H). These values are used to calculate the posterior distribution given the observance of single

or multiple evidences. Any of the commercial spreadsheet programs can facilitate the development of these matrices due to the rapid updating and mathematical capabilites (this thesis used Quattro^{TM}).

P(E:H)

	h1	h2	h3	h4
p1,234	0.3	0	0.257	0.257
p2,134	0.9	0	0.771	0.771
p3,124	0.6	0	0.1	0.9
p4,123	0.05	0.05	0.05	0.05
p12,34	0.27	0	0.198	0.198147
p13,24	0.25	0	0.025	0.2313
p14,23	0.015	0	0.012	0.01285
p23,14	0.54	0	0.077	0.6939
p24,13	0.045	0	0.038	0.03855
p34,12	0.03	0	0.005	0.045
p123,4	0.225	0	0.019	0.178332
p124,3	0.013	0	0.009	0.009907
p134,2	0.012	0	0.001	0.011565
p234,1	0.027	0	0.003	0.034695
p1234	0.011	0	0.000	0.008916

P(-E:H)

	h1	h2	h3	h4
p1,234	0.7	1	0.743	0.743
p2,134	0.1	1	0.229	0.229
p3,124	0.4	1	0.9	0.1
p4,123	0.95	0.95	0.95	0.95
p12,34	0.07	1	0.170	0.170147
p13,24	0.35	1	0.668	0.0743
p14,23	0.665	0.95	0.705	0.70585
p23,14	0.04	1	0.208	0.0229
p24,13	0.095	0.95	0.217	0.21755
p34,12	0.38	0.95	0.855	0.095
p123,4	0.035	1	0.153	0.017014
p124,3	0.088	0.95	0.161	0.161630
p134,2	0.332	0.95	0.635	0.070585
p234,1	0.038	0.95	0.195	0.021755
p1234	0.033	0.95	0.145	0.016163

P((E):H)

	h1	h2	h3	h4
(),1234	0.033	0.95	0.145	0.016163
p1,234	0.00475	0	0.050319	0.005501
p2,134	0.29925	0	0.489780	0.054421
p3,124	0.03325	0	0.016163	0.145475
p4,123	0.00175	0.05	0.007656	0.000850
p12,34	0.04275	0	0.169415	0.018823
p13,24	0.02375	0	0.005501	0.050319
p14,23	0.00025	0	0.002848	0.000294
p23,14	0.29925	0	0.054421	0.489780
p24,13	0.01575	0	0.025778	0.002864
p34,12	0.00175	0	0.000850	0.007656
p123,4	0.21375	0	0.018823	0.169415
p124,3	0.00225	0	0.008916	0.000000
p134,2	0.00125	0	0.000294	0.002848
p234,1	0.01575	0	0.002864	0.025778
p1234,()	0.01125	0	0.000000	0.008916

Figure 5.4. Likelihood Matrices

The $P(E|H)$ and $P(-E|H)$ matrices only concern evidence that is observed to be true/false and does not take into consideration evidence not observed (i.e., in a situation with four evidences, $P(1|H)$ only looks at the probability of Evidence 1 being true without considering the states of Evidences 2, 3, or 4). Likewise, $P(-1|H)$ is Evidence 1 being observed as false given a particular hypothesis, without concern for Evidences 2,3, or 4. The $P(\{E\}|H)$ matrix looks at all the evidence whether it be true or false (i.e., $P(\{1,234\}|H)$ would be interpreted as the probability of Evidence 1 being true and Evidences 2, 3, and 4 being false). The $P(\{E\}|H)$ matrix is made up of the data from the $P(E|H)$ and the $P(-E|H)$ matrices. Because any combination of evidence can be derived from the $P(\{E\}|H)$ matrix, it is the knowledge base that the inference engine of the expert system acts upon when determining posterior probabilities.

5.6.1 Combining Evidence. The ability to incorporate various combinations of the evidence and make rational decisions is an important factor in knowledge reasoning. The decision-analytic model is able to represent combinations of both independent and dependent evidence in a manner that accurately reflects the expert's knowledge.

5.6.1.1 Independent Evidence. In cases where the evidence is independent concerning a certain hypothesis, the individual likelihoods are multiplied together to form joint probabilities. Therefore, the probability of $P(E1,E2|H)$ is the $P(E1|H) \times P(E2|H)$ when E1 and E2 are conditionally independent. This assumption makes it easy to determine likelihoods involving many different evidences. When E1 and E2 are not conditionally independent (see Section 5.6.1.2, *Dependent Evidence*) given the hypothesis, those values are hardwired in the system and are used whenever Evidences 1 and 2 are combined together. Under the assumption that most evidence is independent or nearly enough so for practical purposes, only a small subset of data needs to be hardwired.

5.6.1.2 Dependent Evidence. Because it is unrealistic to assume that all of the evidence is independent, a hypothetical example was generated in which two pieces of evidence are dependent given a certain hypothesis. This demonstrates how the system handles dependent evidence data.

Assume that in this example the cause of an automobile problem is the battery being low (H_1), and evidences E_1 (headlights work) and E_3 (car turns over) are dependent. Initial likelihoods are $P[E_1 \mid H_1] = .3$ and $P[E_3 \mid H_1] = .6$. Assume the joint occurence of both evidences is believed to be .25 (i.e., $P[E_1, E_3 \mid H_1] = .25$). If the evidences were treated as independent events, the joint probability would be .18 ($.3 \times .6 = .18$). This 28% (.25 - .18/.25) difference could have significant ramifications involving decisions based on the joint occurence of these evidences. The tree diagram below (Figure 5.5) graphically shows which numbers need to be accessed anytime these two evidences are combined concerning H_1.

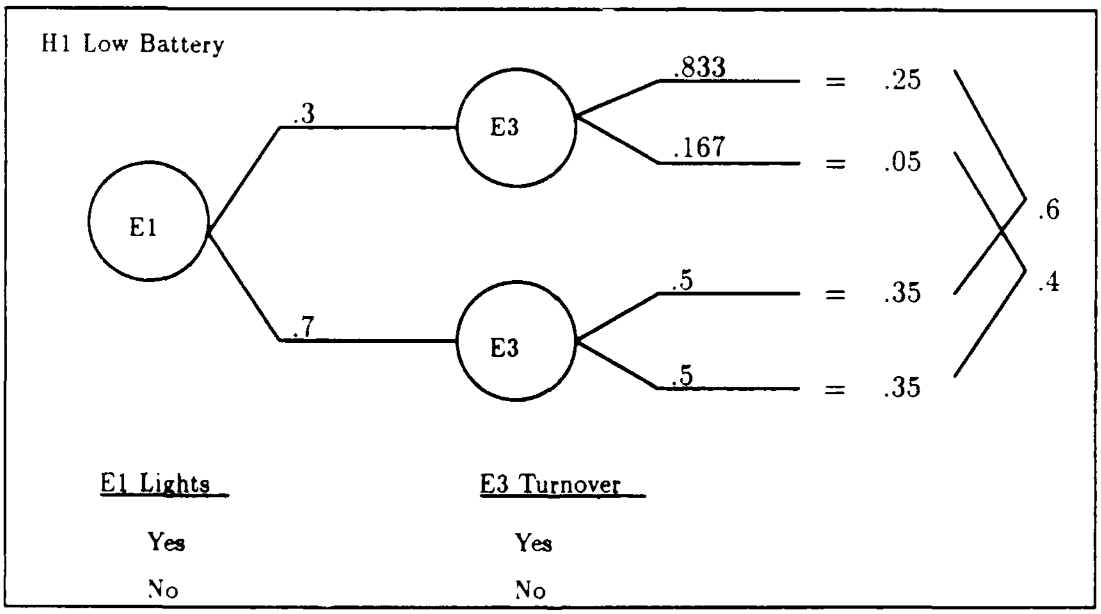

Figure 5.5. Tree Diagram

In this diagram the initial conditional probabilities remain the same. Consequently, if the headlights do work, P[E1|H1] = .3, and if they don't,P[-E1|H1] = .7. If the car turns over, P[E3|H1] = .6, and if it doesn't, P[-E3|H1] = .4. However, the dependency of the evidences requires four additional conditional probabilities to be saved in the system representing joint relationships. These probabilities are as follows:

P[E1,E3| H1] = .25
P[E1,-E3| H1] = .05
P[-E1,E3| H1] = .35
P[-E1,-E3| H1] = .35

The numbers used are hypothetical and are used merely to demonstrate the necessity of accurately capturing the joint likelihoods of dependent evidences. Consequently, any time E1 and E3 are used concerning H1, these numbers need to be used. As for other hypotheses, the probabilities are merely combined as any other independent piece of information.

5.7 *Summary*

This chapter has reviewed the basic outline of the decision-analytic approach used in this thesis. The theory behind Bayesian reasoning was reviewed and how it combined with utility information is a rational decision making tool. The assumptions used in this method have been reviewed as well how this type of expert system offers advantages over traditional rule-based systems because of its added versatility. Why the rules in this model operate and appear like mathematical functions was discussed as well as how the knowledge base is developed. The next chapter discusses model applications.

VI. Generic Formulation

6.1 Introduction

The goal of expert systems is to be able to accurately represent and utilize knowledge in a manner that closely resembles human intelligence. As debated in the literature review, the ability to capture and reason with uncertainty is one aspect of human intelligence that is particularly difficult to grasp. Another heavily debated topic is how should this knowledge be represented. Accepting the statements that (1) probability is the only manner to coherently model uncertainty and (2) rational decision making requires maximizing utility, as being true, this chapter, under the assumptions of Chapter IV, sheds some light on how information can be probabilistically captured in a knowledge base, and how uncertainty assessments by the user are coherently manipulated by a decision-analytic reasoner. As with any expert system, the effectiveness and accuracy of the output is directly related to the knowledge captured in the data base. Consequently, care should be taken to avoid incorrect assumptions, or faulty implications concerning the system description.

6.2 Overview

The ability to accurately represent an expert's interpretation of how knowledge relates together is an important facet of expert system technology. Several prominent papers have been written concerning how this process can be done in a manner that is both easy to apply yet quantitatively and qualitatively accurate. Decision trees, graphs, or networks, including belief networks and influence diagrams, are well known representations which can facilitate the assessment of coherent prior distributions and data dependencies. This thesis did not have to resort to influence diagrams and the like because the expert was able to accurately quantify his knowledge in a probabilistic manner without such tools. However, the domain size in the example was small and no doubt attributed to this result. Given a larger domained problem,

graphical representations such as belief networks and influence diagrams are recommended because of their ease of use and versatility. Current work by Morlan (22) and Deakin (5) shows how these knowledge representation schemes can directly be applied to decision-theoretic models like this one.

Concerning decision-theoretic inference, the assessment of priors, likelihoods, and utility values are important problems.

Priors, in reference to the hypothesis set, give the initial probabilities of each hypothesis without the acquisition of additional evidence. They should reflect the [decision maker's] prior information about the uncertain quantity in question. If this information is primarily in the form of sample results, the priors should be close to the observed relative frequencies. Under the assumption that the hypothesis set is exclusive and exhaustive, the sum of the individual priors must equal one.

Likelihoods are conditional probabilities which capture the probabilistic relationship between evidence and hypotheses. They denote the conditional probability of obtaining a particular sample result (evidence) given some value of the basic random variable (hypothesis). They, as priors, should be based on prior data, if possible.

Utility involves quantifying how useful or valued some action or consequence is. In the assessment of the utility functions, as in the assessment of probabilities, an element of vagueness is involved. However, unless the decision-making problem is highly sensitive to slight changes in the utility function, it appears as if the model is fairly robust over a range of utility values. (Data in this thesis support this assumption.)

6.3 Converting Knowledge into Decision Analytic Format

The following outline shows how GPS contingency procedures can be fashioned into the necessary probabilistic format. The method should be applicable to any type of knowledge that can be written in the "If A then B" structure. The procedure

involves taking data, which in the case of checklists is written in a posterior-type format, and transforming them into likelihood ratios and plausible hypotheses. The following example was taken from GPS OOH 3.7-51.

Data: Disable the loadshed when any of the following conditions are met.

3.7.9.1.2 LS1TM and/or LS2TM > 4.20 v during earth or lunar eclipse.

3.7.9.1.3 LS1TM and/or LS2TM > 4.20 v and IGS is active.

Step 1) Determine state of the world.

- E0 LS1TM > 4.2

Step 2) Determine hypotheses which would explain present state of the world.

H1 - Combined eclipse (earth or lunar).

H2 - IGS is active.

Step 3) Determine relevant evidence (information) needed to discern between hypotheses.

E1 - Vehicle is in eclipse (earth or lunar). (This could be broken down into separate telemetry values or can be looked at as just one evidence state incorporating multiple telemetry values.)

E2 - L3RFP is between 12.6 and 22.5.

Step 4) Determine appropriate response actions for each hypothesis.

A1 - Disable loadshed.

After putting information into the necessary format, subjective calculations of likelihoods, priors, and utility actions are made. This would include the joint likelihood matrix and the action diagram. Below are hypothetical assessments.

- $P(E1|H1,E0) = 1.00$
- $P(E2|H1,E0) = .00001$
- $P(E1|H2,E0) = .0001$

- $P(E2|H2.E0) = 1.00$
- $P(H1) = .990$
- $P(H2) = .010$

The data is now in the necessary format to be accessed by a Decision-Analytic Reasoner (see Appendix C, Scheme Program). Upon the observation of the LS1TM and/or LS2TM being greater than 4.2V, this area of the knowledge base would be accessed. In this case, there are only two relevant evidences which need to be observed. Upon the observation of these telemetry points, the appropriate action would be recommended. In this case, the only action is to disable the loadshed timer. This is a simple example which demonstrates how "If$\cdots$then$\cdots$" information can be captured probabilistically in the knowledge base.

6.4 Likelihood Determination

In order to determine the appropriate likelihood values to be used in determining the posterior probabilities, the $P(\{E\} \mid H)$ table is referenced (see Figure 6.1). This table contains the set of joint likelihood values which were determined by the expert. For example, assume Evidences 1, 2, and 3 were observed, with E12 being true and E3 being false. The table is arranged so that all affirmative evidences are on the left and all negative responses to the evidence are on the right, separated by a comma (e.g., $P(E12,3|H_i)$).

P((E);H)

	h1	h2	h3	h4
(),1234	0.033	0.95	0.145	0.016163
p1,234	0.00475	0	0.050319	0.005591
p2,134	0.29925	0	0.489789	0.054421
p3,124	0.03325	0	0.016163	0.145475
p4,123	0.00175	0.05	0.007656	0.000850
p12,34	0.04275	0	0.169415	0.018823
p13,24	0.02375	0	0.005591	0.050319
p14,23	0.00025	0	0.002648	0.000294
p23,14	0.29925	0	0.054421	0.489789
p24,13	0.01575	0	0.025778	0.002864
p34,12	0.00175	0	0.000850	0.007656
p123,4	0.21375	0	0.018823	0.169415
p124,3	0.00225	0	0.008916	0.000990
p134,2	0.00125	0	0.000294	0.002648
p234,1	0.01575	0	0.002864	0.025778
p1234,()	0.01125	0	0.000990	0.008916

Figure 6.1. Joint Likelihood Matrix

However, the data base table is formulated with all evidences being observed given a particular scenario, and, in this case, we have only observed three of the possible four. In order to determine the necessary likelihoods considering only the three observed evidences, all combinations of the data where E12 are on the left and E3 is on the right are summed together. This would involve the following combinations of data: (E124,3) and (E12,34) (see Table 6.1. Assuming there is no uncertainty in the evidence states, the likelihood value would be:

Combined with the prior probabilities, these values would be used in determining the posterior distribution.

Table 6.1. Determining Likelihood With No Uncertainty

	H1	H2	H3	H4
(12,34)	0.04275	0	0.169415	0.018823
+				
(124,3)	0.00225	0	0.008916	0.000990
(12,3)	0.045	0	0.178331	0.019813

6.5 *Uncertainty Assessment*

In order to modify the evidence according to the subjective belief assessment, a derivation of Bayes' Rule (see Chapter IV) is used. A vector is created which enumerates the various combinations of the data incorporating the belief assessments. In cases where the belief is some number other than one, both cases of confirmation and disconfirmation are factored into the assessment. Consequently, if belief in some evidence is .7, then the disbelief in that evidence must be .3. Table 6.2 below shows the vector created when the belief in Evidences 1 and 2 are .9 and .1, respectively. This value is the likelihood value used in determining the posterior probabilities.

P(E12,3|Hi)

Table 6.2. Determining Likelihood With Uncertainty

Vector			*Posterior (Vector)*	
E1	E2	E3		
Y(.9)	Y(.1)	N(1.0)		P(Hi|E12,3)*P(.9*.1*1.0)
Y(.9)	N(1-.1)	N(1.0)	+	P(Hi|E1,23)*P(.9*.9*1.0)
N(1-.9)	Y(.1)	N(1.0)	+	P(Hi|E2,13)*P(.1*.1*1.0)
N(1-.9)	N(1-.1)	N(1.0)	+	P(Hi|E,123)*P(.1*.9*1.0)

6.6 *Posterior Assessment*

Using Bayes' Rule, posteriors are determined based upon the evidence observed and the degree of belief. Posterior values involving each of the possible hypotheses

are calculated and these values are combined with the utility values of the action matrix. The DA reasoner does a comparison of the resultant utilities and using the rational approach of maximizing utility, recommends the action yielding the highest result. See Figure 6.2 below showing a 2 hypothesis and 2 action decision matrix. In this case the selection is made according to the following formula for expected utility:

If $P(H1|E)a + P(H2|E)b > P(H1|E)c + P(H2|E)d$
Then A1 is preferred to A2.

	H1	H2
A1	a	b
A2	c	d

Figure 6.2. Action Utility Table

6.7 Action Vectors and Utility Assessments

The action matrix ties together all actions which have been identified as producing definite benefits (or losses) if taken when the hypothesis is true.

Given the plausible set of hypotheses, which can explain the present state of the world, actions are determined which would provide the most utility given perfect information. In so doing, there are as many optimal actions as there are hypotheses.

This assumes that each hypothesis would have a different optimal action associated with it.

Using a reference lottery, and a range from 0–100, the various actions of the plausible set are scaled in relation to one another as to the amount of utility they provide given each hypothesis. It is assumed that the optimal action for each hypothesis is given a value of 100. This represents doing the best that can be done under the present circumstances. A value of zero conversely would represent an action that not only did not help the situation, but could also be potentially damaging. As mentioned before, this ranking within each hypothesis is also a subjective process. Raiffa (25), gives a good description of how these values can be determined using a reference lottery.

6.8 Determining Utility Assessments Between Hypotheses

After utility data is determined within a particular hypothesis for the various actions, there must be some way of relating these assessments together so that they adhere to the expert's overall belief in the hypothesis' importance. This thesis used a ratio formula based on criticality to determine relative weights. This method, described below, is by no means the only way to determine this scale ranking.

Given the plausible set of hypotheses, the expert needs to rank order them from the most critical to the least. The least critical hypothesis is assigned a value of 1 and then the values of the others are determine by means of ratios. In the example used in the thesis concerning the loadshed 1 timer (see Chapter VII), the hypotheses were ranked as follows:

1. Attitude problem,
2. Solar array not tracking,
3. Faulty timer,
4. Eclipse operations.

The eclipse, which is a routine and expected event, was assigned a value of 1 and the the expert was queried as to how many times she would be willing to make a wrong assessment concerning the eclipse versus an attitude problem. The value was 100 times for the attitude problem, 20 times for a bad timer, and 50 times for a solar array tracking problem. Figure 6.3 shows the unscaled utility values initially provided by the expert and the scaled values which incorporate the ratios. The scaled utility table is the one captured in the knowledge base and one used by the DA reasoner.

Unscaled

	H1	H2	H3	H4
A1	100	2	50	10
A2	0	100	0	10
A3	40	10	100	60
A4	10	40	10	100

Scaled

	H1	H2	H3	H4
A1	100	200	1000	500
A2	0	10000	0	500
A3	40	1000	2000	3000
A4	10	4000	200	5000

Figure 6.3. Utility Table

The assessment of utility, like most probabilistic inferences, is a subjective process. As with probability assessments, much has been done in the area of detemining how to accurately capture an expert's value judgements concerning utility assessments. Studies show that unless the decision making problem is highly sensitive to slight changes in the utility factors, it shouldn't be too difficult to determine a set of utilities that are reasonably good approximations of the expert's preference.

6.9 Summary

This chapter provided insight as to how knowledge can be captured in a probabilistic manner for use in a decision-analytic system, how uncertainty in the evidence is represented using Jeffrey's Rule, and how utility values are assigned. Also discussed were the advantages this approach offers. Though this thesis deals with manipulating probabilities, it has not concerned itself as to the various methods with which these probilities can be ascertained. Several papers advocating probabilistic representation of knowledge have deeply researched this area with algorithms and other heurestic techniques which can be used to obtain accurate probabilities from expert knowledge. Because of the assumptions that this method is best applied towards areas which already lend themselves to probabilistic methods due to large historical data bases, known frequencies, etc., discussions concerning specific techniques to obtain probabilities in complex domains were not presented. This chapter has laid the groundwork for the next chapter in which the method will be applied to an actual problem.

VII. Problem Application

7.1 Introduction

The following example was generated as a test case to demonstrate the utility of the decision-analytic approach to rule-based expert system development. The example concerns satellite anomaly resolution, and serves as a guideline as to what information is required in the application of the methodology. Given the fact that on-orbit anomalies in military satellite systems or subsystems occur quite often, and the number, frequency, and severity of the anomalies are likely to grow with the inevitable increases in spacecraft complexity, (17:207) the successful application of this approach to this type of problem should be of interest to the Air Force.

7.2 Assumptions in Satellite Anomaly Resolution

The following assumptions apply to fault diagnosis in satellite operations.

(a) Time between failures is long compared to diagnosis time, ruling out simultaneous failures (which, historically, do not happen in this enviroment).

(b) Sensors, which are monitored via telemetry can either be the cause of an anomaly or merely indicate one has occured. Therefore, information from adjacent values can be used to determine the veracity of individual sensor readings.

(c) Past trends can be used to reliably forecast future problems.

7.3 Problem Formulation

The situation concerns a hypothetical, yet realistic, occurence in GPS satellite operations in which the Loadshed One Timer on the vehicle is unexpectedly timing up. Based on previous experiences and historical telemetry data, GPS Electrical Power Subsystem (EPS) engineers have concluded that there are four likely conditions (hypotheses) that could explain the sensor reading.

These conditions are:

1. Eclipse operations;
2. Attitude problem (loss-of-earth, loss-of-yaw control);
3. The timer is faulty; or
4. The solar arrays are not tracking.

Based on these hypotheses, the appropriate actions are either going to be

1. Monitor the vehicle;
2. Carry out appropriate attitude contingency procedure;
3. Disable the loadshed; or
4. Perform solar array contingency procedures.

In order to ascertain the problem and narrow the hypothesis set, the following telemetry values (evidence) can be consulted:

1. Loadshed 2 is also timing up;
2. SDV (shunt dissipation voltage) $> .3$;
3. BCC (Boost Convert Current) $< .15$;
4. $\pm$ YSAC (Solar Array Current) summed is > 1.0;
5. Yaw1 and Yaw2 > 1.5 and HZ $> \pm .35$;
6. Pitch1 and Roll1 > 0.5 degrees; and
7. YER (Yaw pointing error) > 2 degrees or YPOS of one wing is $> \pm 5$ degrees.

These values correspond to critical points the GPS Orbital Operations Handbook (OOH) Sections 3.7 and 3.8 enumerates. The value tolerance levels have been determined from these contingency procedures as well as from expert input. As

with the initial assumptions previously mentioned, the hypotheses form an exclusive and exhaustive set (i.e., the problem is definitely one of these choices, and it is not a combination of two or more hypotheses). Also, only one action will be recommended. In real world satellite operations most decisions concerning anomaly resolution are made in group decisions. So, even though the expert system would recommend a particular course of action, more often than not this decision would be reviewed before any action is initiated. For purposes of this thesis, the goal will be the recommendation of an action based on the available evidence.

In order to facilitate testing of the methodology a computer program coded in PC Scheme was developed as a prototype decision-analytic reasoner. The program, using the Quattro^{TM}-generated file as the knowledge base, allowed for varying degrees of evidence belief to be manipulated in a manner which adhered to Bayes' theorem, while also calculating utility values in a decision theoretic manner.

7.4 Knowledge Base

The following likelihoods and priors were determined from personal interviews and phone conversations with Lt Pam Neal, Master Control Station (MCS) EPS Engineer. The values are based on the scenario that Loadshed 1 Timer is increasing (E_0). Suggested actions were determined from the GPS OOH as well as inputs from Lt Neal.

7.4.1 Priors. The prior probabilities of the hypotheses were determined mathematically. Given the initial scenario, Lt Neal ranked the likelihood of possible outcomes in relation to one another. It was determined that H2 (attitude problem) was the least likely and each of the other values was determined in relation to it. It was determined that the likelihood of eclipse operations to an attitude problem was 500 to 1; a faulty timer to an attitude problem was 1 to 1; and a solar array tracking problem to attitude problem 10 to 1. Since the hypotheses are mutually exclusive

and exhaustive, the probabilities are summed and the unknowns set equal to 1. The solved equations yield the following prior probabilities:

$P[H1] = .9766$
$P[H2] = .001953$
$P[H3] = .001953$
$P[H4] = .01953$

7.4.2 Likelihoods. The following likelihood data was determined by EPS engineers to be an accurate representation of the data, given the initial scenario of Loadshed 1 timing up.

```
Priors              P[Ei|H1] Eclipse     P[Ei|H2] Attitude problem

P[H1] = .9766       P[E1|H1] = .999      P[E1|H2] = .999
P[H2] = .001953     P[E2|H1] = .005      P[E2|H2] = .005
P[H3] = .001953     P[E3|H1] = .001      P[E3|H2] = .001
P[H4] = .01953      P[E4|H1] = .9        P[E4|H2] = .99
                    P[E5|H1] = .81       P[E5|H2] = .92
                    P[E6|H1] = .0001     P[E6|H2] = .81
                    P[E7|H1] = .45       P[E7|H2] = .58

                    P[Ei|H3] Faulty Timer P[Ei|H4] Solar Array Prob

                    P[E1|H3] = .001       P[E1|H4] = .999
                    P[E2|H3] = .989       P[E2|H4] = .005
                    P[E3|H3] = .999       P[E3|H4] = .001
                    P[E4|H3] = .9         P[E4|H4] = .99
                    P[E5|H3] = .0001      P[E5|H4] = .2
                    P[E6|H3] = .0001      P[E6|H4] = .02
                    P[E7|H3] = .001       P[E7|H4] = .99
```

These values were used to generate the joint likelihood matrix. See Appendix E for Quattro^{TM} data files.

7.4.3 Utilities and Actions The following utility values were determined as accurate representations of the expert's belief of the relative utility each action provided given the a particular hypothesis (see Figure 7.1). Diagonal values of 100 indicate doing the correct action given the stated hypothesis. Lower values represent the relative utility provided by an action given an inaccurate assessment of the problem. The scaled utility table, which reflected the relative importance of the hypotheses, was used in the knowledge base (see Section 6.8) for the actual problem application.

	H1	H2	H3	H4
A1	100	2	50	10
A2	0	100	0	10
A3	40	10	100	60
A4	10	40	10	100

Figure 7.1. Unscaled Utility Table

7.5 Validation and Testing of Methodology

The decision-analytic methodology was tested and evaluated using the GPS OOH, Sections 3.7 - 3.8 which concern satellite anomalies, as well as from discussions with the MCS GPS EPS satellite engineer. Comparing the model's output against

the expert's recommended actions and contingency procedures in the OOH proved to be quite successful.

7.5.1 Test Methodology. In order to test the viability of accurately representing GPS system knowledge, several computer runs were done with a PC Scheme-coded program which combines information in a decision-theoretic manner. This program was developed as a prototype inference engine which reasoned with uncertainty using a derivative of Bayes' Rule and made recommendations according to maximum utility. (See Appendix C for computer code.) The utility and likelihood values provided by the expert were used as the knowledge base.

Simulating the situation of seeing the timer increase, the expert was asked; given the observance of certain telemetry points (no uncertainty involved), what would he surmise the problem to be and what course of action would be taken. Because the evidence set is a compilation of the evidences which have been determined as necessary to differentiate between the different hypotheses, conflicting subsets of the evidence data were not mixed. This would avoid giving credance to multiple problems occuring simultaneously, which would violate one of the assumptions of this thesis. This testing was done in order to confirm that the assumptions used in combining the data (i.e., independence) would, in fact, identify highly suspected problems. It was also used to validate the utility values and likelihoods in order to see if they did indeed provide answers which coincided with expert opinion and GPS OOH material.

The system performed quite well, correctly identifying suspected problems in all cases where all seven of the telemetry points in the evidence set were commented upon. See Appendix F on computer runs to show actual output versus expected output. However, some interesting facts were observed which showed that the system had apparently captured some heuristic expert knowledge that would not show up in the GPS OOH. In particular, when Evidences (1 and 6) are observed as true, and

Evidences (2 3 4 5 7) are observed as false, the correct assessment is an attitude problem with the recommendation to start attitude contingency procedures. The DA reasoner coincided with this assessment outputting a normalized utility value of about 63% in favor of an attitude problem. However what is interesting is the fact that if the belief in the level of E-6 drops below .995, the suspected hypothesis is an eclipse (normal operations). The expert agreed this was an accurate assessment due to the fact that an attitude problem is so rare, you would almost have to be 100% sure of the telemetry value before you would act accordingly. This stands in direct conflict with the GPS OOH, which, written under the guise that all telemetry values are assumed to be true, recommends that appropriate actions be taken regardless of an operator's subjective feelings. In order to further illustrate the manner in which the model had mimmicked the expert's reasoning, if the belief in E-6 drops to .9, the normalized belief in an attitude problem drops to only 3%. See results below for corresponding computer output.

This shows how certain one must be of the loss of earth values before appropriate action would be taken.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(6 1))((2 1)(3 1)(4 1)(5 1)(7 1)))
                          ------------------------
2.2255475021483e-4       |0.026347394230459      |
0.00528862635          * |0.62609997426044       |
6.6109466042966e-4       |0.0782644343703208     |
0.00227466014004297      |0.269288197138781      |
                          ------------------------
Do action 2
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(6 .996))((2 1)(3 1)(4 1)(5 1)(7 1)))

                         ------------------------
0.00427808700996717     |0.293918175243603      |
0.0052754674266       * |0.362441379983816      |
0.00229851447977434     |0.157915250459447      |
0.00270329842840143     |0.185725194313134      |
                         ------------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(6 .9))((2 1)(3 1)(4 1)(5 1)(7 1)))

                         ------------------------
0.101610861244023       |0.630505693315243      |
0.004959653265        * |0.0307751512207163     |
0.0415965901440467      |0.25811105808204       |
0.0129906173490047      |0.0806080973820009     |
                         ------------------------
Do action 1
```

7.5.2 Uncertainty Testing. Varying the belief in the evidence (uncertainty) proved to be quite interesting, especially when conflicting information was entered. The expert really didn't know how to approach this type of problem because of the complexity of trying to determine which problem to act on or how to logically combine several pieces of partial information. This is precisely the type of situation in which expert systems can be used to augment operator knowledge by recommending actions which will maximize the expected utility.

Given the expert's inability to quantify her actions given certain hypothetical situations, several runs were done showing exactly how sensitive the system was to varying degrees of belief. Though several cases were done in which the belief in the evidence ranged from .1 to 1, the expert noted that given the reliability of the satellite equipment, it is unlikely that belief would ever drop below .8. Also, due to

the fact that a .5 belief in a value is also a .5 disbelief in it (indifference), it would not make sense logically to use values below this mark. Appendix G is a listing of a few computer runs which show how slight variations in the levels of uncertainty can cause a small change in the output utility values or a large change. Because of the rational nature in which the system operates, the actions recommended will maximize the expected utility.

7.5.3 Sensitivity. Sensitivity testing was done in order to see exactly how robust the model was given the variability of the prior and utility information. Three scenarios were involved in the testing: the first concerned a suspected attitude problem, the second concerned a faulty timer, and the third involved a solar array problem. The eclipse operations hypothesis was not tested because under the scenario that the data was developed, this would indicate normal operations.

7.5.4 Prior Sensitivity. In the prior test, three sets of prior values were tested with the scaled utility data in order to determine how sensitive the output was to change. First, the expert provided data was used as a gold standard in order to measure any noticeable change given the variability of the data. The first test involved modifying the ratios provided the expert which were used to calculate the initial priors. Instead of predicting that the eclipse operations are 500 times more likely than the attitude problem, the number was halved to 250. The ratio of the solar array problem was doubled from 10 to 20 times more prevalent while the ratio between the attitude and faulty timer problems stayed the same. Using these ratios, the new prior values are:
H1 = .9191, H2 = .00367, H3 = .00367, and H4 = .07353. See Section 2.4.1 for formula used to solve equation. Test Two involved changing the prior values to (.75 .05 .05 .15) while the Test Three set all values equal to .25.

These tests concern varying the priors with Scenario 1, the attitude problem. This first test is the gold standard with the expert's priors and utility values.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(5 1)(6 1))((2 1)(3 1)(4 1)(7 1)))
                          -----------------------
0.00164951870500002   |0.0177405542122364     |
0.060703116705        |0.652861304023074      |
0 00626584711000004   |0.0673891117473198     |
0.02436162877         |0.26200903001737       |
                          -----------------------
Do action 2
```

This test is with prior values of (.9191 .00367 .00367 .07353) Though the values have been modified accordingly, the effect is minimal with all values maintaining roughly the same percentages.

```
                          -----------------------
0.00270210990500004   |0.0155071975041286     |
0.114078195705        |0.654685846951901      |
0.0116564311100001    |0.0668953468848772     |
0.04581202127         |0.262911608659094      |
                          -----------------------
Do action 2
```

This test is done using prior values of (.75 .05 .05 .15) and scaled utilities. A more dramatic change of the priors still results in minimal change of the values. The attitude problem still has about 66% of the utility yet its prior is 25 times higher than it originally was.

```
                          -----------------------
0.0314412750000006    |0.0133050227978924     |
1.554029775           |0.657619692107851      |
0.155711250000001     |0.0658924210655677     |
0.6219309             |0.263182864028689      |
                          -----------------------
Do action 2
```

This test was done with all the priors equal to (.25 .25 .25 .25) Even with equal priors, no change is really noticeable in this scenario. I attribute this to both the likelihood values and the high utility values associated with the attitude problem.

```
                       -------------------------
0.155560125000003     |0.0131702717674423       |
7.770049625           |0.657839952287012        |
0.777341950000006     |0.0658125257853416       |
3.1085073             |0.263177250160204        |
                       -------------------------
Do action 2
```

These prior tests were done on Scenario 2, the faulty timer. This is with the expert provided priors and scaled utilities. The timer has 62.2% of the utility while the eclipse has 31%, the attitude problem zero percent and the solar array 6%.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1)(3 1))((4 1)(5 1)(6 1)(7 1)))

                       -------------------------
1.978629044814e-4     |0.316803337336471        |
1.005795e-8           |1.61040399923114e-5      |
3.87584931087e-4      |0.620572107700108        |
3.9102776028e-5       |0.0626084509234294       |
                       -------------------------
Do action 3
```

This test was done with priors of (.9191 .00367 .00367 .07353) With the modified priors, the values stay essentially the same.

```
                       -------------------------
3.67041348026e-4      |0.314623078595486        |
2.611918e-8           |2.23890220166899e-5      |
7.26464722810001e-4   |0.622716129424492        |
7.30743719200001e-5   |0.062638402958005        |
                       -------------------------
Do action 3
```

This test was done with priors of (.75 .05 .05 .15). This dramatic change in the priors still results in roughly the same proportional utilities.

0.00493894759000001	0.312618446635 89
1.889e-7	1.19567222557872e-5
0.00987175835000001	0.624848452976168
9.87749300000001e-4	0.0625211436656868

Do action 3

This test was done with priors of (.25 .25 .25 .25) Even with equal priors. the timer problem is still identified with 62% of the utility. Just like the attitude problem it appears as if the likelihood values combined with the utilities are determining the result.

0.02467636995	0.312501314443063
8.465e-7	1.07200679521362e-5
0.0493508957500001	0.624979274588059
0.0049359395	0.0625086909009255

Do action 3

This is Scenario 3, the solar array problem, with the experts original priors and the scaled utilities. The solar array problem has 52% of the utility with the eclipse at 10%, the attitude problem at 5% and the timer at 33%.

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(7 1))((2 1)(3 1)(5 1)(6 1)))

14.9160660502193	0.102468532262818
7.62023493	0.0523485405700168
47.7074220910387	0.327734504744941
75.3235620340039	0.517448422422224

Do action 4

This was done with priors of (.9191 0.00367 0.00367 0.07353) The slight change in the priors caused the eclipse to become the recommended action with a value of 55%. The solar array goes to 44% while both the attitude and the timer drop to zero.

```
                         ----------------------
0.0702238355             |0.555739383545053    |
3.180789e-5              |2.51722183139211e-4  |
3.6333e-14               |2.87533127157978e13  |
0.05610544884            |0.44400889427152     |
                         ----------------------
Do action 1
```

This is with the priors of (.75 .05 .05 .15). With these priors, once again the solar array is the recommended action with 66% of the utility. The eclipse drops from 55% to 33%.

```
                         ----------------------
0.05730375               |0.332791203735817    |
4.3335e-4                |0.0025166776718612   |
4.95000000000001e-13     |2.87470969786846e-12 |
0.1144542                |0.664692118589447    |
                         ----------------------
Do action 4
```

This is with equal priors of (.25 .25 .25 .25). With equal priors the selection of the solar array problem is pronounced with almost 90% of the utility. Eclipse goes from 33% to 9% while attitude increases from zero to 1.

```
                         ----------------------
0.01910125               |0.900896120729963    |
0.00216675               |0.0102193137600505   |
2.475e-12                |1.16731517508365e-11 |
0.190757                 |0.89969107415528     |
                         ----------------------
Do action 4
```

7.5.5 *Utility Sensitivity.* In order to test the effects of utility values on the recommendations of certain actions given the evidence, three tests were done in each scenario. Test One involved the expert provided data in which the scaled utility values were used. This served as the standard with which other values could be

compared. Test Two involved using the unscaled utility values which were provided by the expert to see if the scaling actually had any noticeable effect on the output. Lastly in Test Three, the posterior probabilities would be examined in order to see what if any different action would be pursued given the knowledge of the likelihood and prior data without utility data. As in the prior test, these tests were run on scenarios that would indicate an attitude problem, a faulty timer problem and a solar array problem.

These tests concern varying the priors with Scenario 1, the attitude problem. This first test is the gold standard with the expert's priors and utility values. In this case the attitude action has a 65.2% value associated with it.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(5 1)(6 1))((2 1)(3 1)(4 1)(7 1)))

                      ------------------------
0.00164951870500002  |0.0177405542122364     |
0.060703116705       |0.652861304023074      |
0.00626584711000004  |0.0673891117473198     |
0.02436162877        |0.26200903001737       |
                      ------------------------

Do action 2
```

This is with the unscaled utility values provided by the expert. In this case the belief in the attitude problem drops from 65% to about 39% while the eclipse action (1) has climbed from 1.7% to 28%. The solar array problem dropped from 26% to 18% and the timer increased from 6% to 14%.

```
                      ------------------------
4.43874582100001e-4  |0.282451031895904      |
6.070699341e-4       |0.386297247542986      |
2.33827324600002e-4  |0.14879150954368       |
2.86738021e-4        |0.18246021101743       |
                      ------------------------

Do action 2
```

This is with only the posterior values used. No utility information. Though the attitude problem is still recommended with an increased value of 58%, the strong prior associated with the eclipse hypothesis (.9766) causes this value to go up to 41% while the timer and solar array problems went to zero.

```
                         ------------------------
4.316572e-6              |0.415284628041265      |
6.069924e-6              |0.583969139309422      |
2.1483e-20               |2.06681590489178e-15   |
7.75341e-9               |7.45932649310941e-4    |
                         ------------------------

Do action 2
```

This is Scenario 2, the faulty timer. Values used are those provided by the expert. In ths case the timer value is 62%, the eclipse 31%, the attitude problem essentially zero, and the solar array about 6%.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1)(3 1))((4 1)(5 1)(6 1)(7 1)))

                         ------------------------
1.978629044814e-4        |0.316803337336471      |
1.005795e-8              |1.61040399923114e-5    |
3.87584931087e-4         |0.620572107700108      |
3.9102776028e-5          |0.0626084509234294     |
                         ------------------------

Do action 3
```

This is with the unscaled utility values. In this case the timer goes down to 55% while the eclipse increases from 31% to 38%. The attitude problem goes even lower while the solar array stays about the same.

```
                         ------------------------
1.4735984803614e-5       |0.382852028280071      |
1.388583e-10             |3.60764363610666e-6    |
2.131571637567e-5        |0.553798429994754      |
2.43818669628e-6         |0.0633459340815384     |
                         ------------------------

Do action 3
```

This is with only the posterior values. The timer is identified with a strong 79% value. I attribute this to these 3 evidences being very descriptive of a faulty timer anomaly. The eclipse drops to 21% while both the attitude and solar array problems are zero.

```
                      -----------------------
5.09785200000001e-8  |0.209144449071664     |
6.23007e-13          |2.55594818725201e-6   |
1.927611e-7          |0.790821586463239     |
7.65576e-12          |3.14085169091783e-5   |
                      -----------------------
Do action 3
```

This is Scenario 3, the solar array not tracking. Values used are those provided by the expert. In this case the the distribution is rather spread out. The solar array problem holds just over half the utility with 52%. The timer is next with 33% while the eclipse and attitude problems are 10% and 5%, respectively.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(7 1))((2 1)(3 1)(5 1)(6 1)))

                      -----------------------
14.9160660502193     |0.102468532262818     |
7.62023493           |0.0523485405700168    |
47.7074220910387     |0.327734504744941     |
75.3235620340039     |0.517448422422224     |
                      -----------------------
Do action 4
```

This is with the unscaled utility values. Without the scaling, the eclipse scenario is now the choice with 55% while the solar array problem goes to 16%. This is quite a dramatic turnaround. I attribute this to strength of the prior for the eclipse and also because Evidences 1 and 4 also lend belief to an eclipse. The attitude problem drops to 1% while the time- problem goes to 28%.

```
                      -----------------------
7.61076552170296     |0.548425223791249     |
0.1507120335         |0.0108601796316775    |
3.87897039691193     |0.279515273718491     |
2.23704198004019     |0.161199322858582     |
                      -----------------------
Do action 1
```

This is with only the posterior values. The dominant prior of the eclipse causes it to be selected with 83% of the total utility. The solar array problem is still about the same (16%) while both the timer and the attitude problem drop to zero.

```
                           -----------------------
0.074617123               |0.833375784690598     |
1.6926651e-5              |1.89048578826992e-4   |
1.93347e-14               |2.15943340300822e-13  |
0.01490193684             |0.166435166730359     |
                           -----------------------
Do action 1
```

7.5.6 Stength of Likelihoods I believe that the very minor changes observed in some of the output given the variability of the prior and utility data indicates the strength that is contained in the likelihood data. I also attribute this lack of variation to the dissimilarity of the problems and the discriminating evidences. If you examine the cases below you can see that in all three cases, given the observed values, the likelihood values are in the 90% range of the suspected problem with the attitude and faulty timer values being 99%. Consequently the utilty values would have to be ridiculously large or the prior values so insignificant in order to effectively overcome the strength of the likelihood. Had there been two problems in which the evidences were the same yet the likelihoods were slightly different, the effect of varying the priors or utilities would have a much more pronounced effect. This confirms my belief that the strength of the implication in the rules which operate in this type of system are contained in the likelihood data.

This is the likelihood value of the attitude problem.

```
                           -----------------------
1.105e-6                  |0.00141993570454029   |
7.77e-4                 * |0.998452527083985     |
2.75e-18                  |3.53377664025863e-15  |
9.925e-8                  |1.27537211471153e-4   |
                           -----------------------
Do action 2
```

This is the likelihood values for the faulty timer problem.

```
                           ------------------------
1.305e-8                   |5.28592012847095e-4    |
7.975e-11                  |3.23028452295446e-6    |
2.4675e-5                * |0.999464208199392      |
9.8e-11                    |3.96950323823871e-6    |
                           ------------------------
Do action 3
```

This is the likelihood values for the solar array problem.

```
                           ------------------------
0.01910125                 |0.0900896120729963     |
0.00216675                 |0.0102193137600505     |
2.475e-12                  |1.16731517508365e-11   |
0.190757                 * |0.89969107415528       |
                           ------------------------
Do action 4
```

The robustness of the system can be attributed to the combination of the likelihood, priors, and utility values. Given the fact that the expert provided values are fairly accurate, varying the prior or utility information still will usually result in the expected outcome. Further testing would have to be done in order to see if this robustness would hold given varying degrees of belief in the evidence or changes in the likelihood data.

The expert noted that determining the likelihoods was the easiest of all the subjective evaluations. Consequently, I feel fairly safe in attributing the overall robust nature of the system to the strength of the likelihoods. Judging from this example, it appears that if the likelihoods give an accurate representation of the data, the priors and the utility values can compensate for the inaccuracy of one another allowing the system to still yield favorable results. Further testing in different situations would have to be done before this hypothesis could really be substantiated.

7.6 *Functional Relationship of Evidences*

Because the DA model represents and manipulates the system knowledge in functional in manner, a few evidences were isolated and graphically depicted in order to examine this relationship. This type of information can be used in sensitivity analysis to determine which values have the most effect on the action. Looking at the two telemetry points E1 and E3, (Figure 7.2) shows how these values relate functionally in the decision space. (Figure 7.3) shows the complex relationship between E123 and E7. Points falling along the lines separating the different action spaces indicate values where the user would be indifferent between the two actions. Although Morlan (13) recommends determining these functions and writing rules which represent their information, this research did not because of the complexity involved with accurately defining such functions and because of the limited flexibility the system would have due to the complexity of altering any of the values. Furthermore, though this might be possible in two-space given the ability to graph the function, as the number of evidences increased beyond three, the ability to graphically represent the decision space would dissappear.

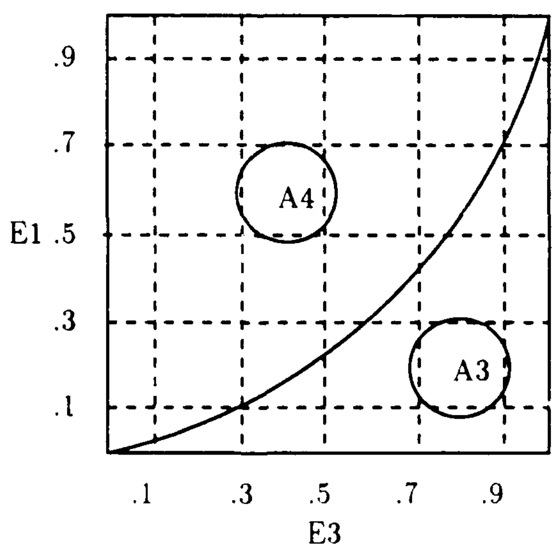

Figure 7.2. Graph of E1 vs E3

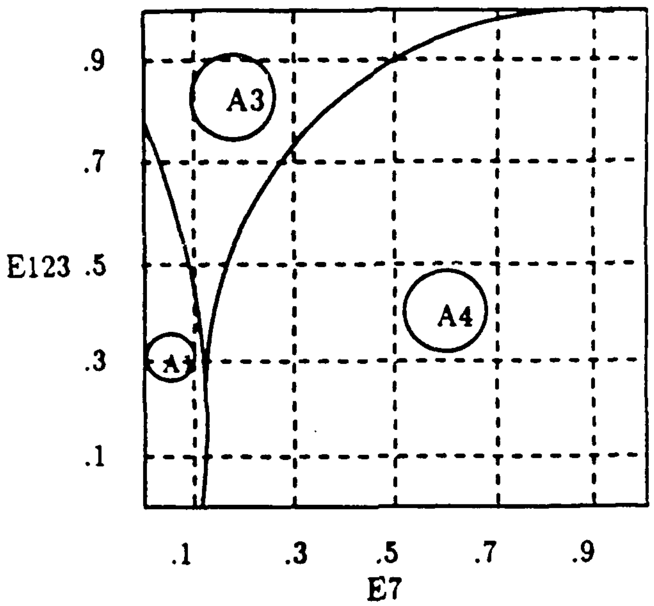

Figure 7.3. Graph of E123 vs E7

7.7 Summary

Comparing the output of the computer program given the data provided by the expert showed that it is possible to accurately model an expert's knowledge in a probabilisitic manner. The output provided by the decision-analytic reasoner not only agreed with checklist procedures in the GPS OOH, but also demonstrated the ability to reason in a manner which matched the expert's given the validity of the evidence. The mathematical nature in which the model represented the rules provided the ability to graphically depict the functional relationship of the evidences showing which were the most sensitive, as well as which were the most robust given certain values. Though expected predictions were difficult to make given uncertainty of the evidence, I operated under the premise was that if the system provided rational output which agreed with the expert's assessment under certainty, it would also

provide rational decisions under uncertainty or with conflicting information. The robustness inherent in a decision-analytic model is no doubt a function of the prior, likelihood, a utility data. In this case due to the dissimilarity of the hypotheses, the likelihood data played the dominant factor in determining actions. However, given the domain and the scenario in which this model was developed this was to be expected.

Though the decision-analytic reasoner developed to test the application of the methodology is functionally correct, it nonetheless is a rough prototype used merely to show the capability of using decision theory as a manner of capturing system knowledge while also reasoning coherently and consistently with uncertainty.

VIII. Conclusions and Recommendations

8.1 Introduction

The purpose of this thesis was to demonstrate the applicability of using a decision theory approach towards reasoning in expert systems. Particular emphasis was placed on the management of uncertainty and how it could be consistently factored into a decision. Using a Quattro^{TM}-generated file as a knowledge base and a Scheme pc-coded program as a decision-analytic reasoner, this capability was demonstrated using an anomalous condition onboard the GPS satellite as the test scenario. The model was able to effectively capture GPS system knowledge probabilistically in a manner that matched expectations of the expert as well as coincided with the GPS OOH. Though the application is complex, it is nonetheless a valid method for building an expert system, given the application domain satisfies the assumptions and constraints under which the model is based.

8.2 Key Concepts

Probability was used as the representation for uncertainty because of its solid theoretic basis, with a derivative of Bayes' Rule being used in the actual problem application. This allowed coherent manipulation of the evidence belief values in a manner that adhered to the laws of probability, yet allowed for subjective interpretation. Utility assessments allowed the user the ability to rank the importance of certain actions in terms of his own value scale. Combined together, the two areas provided a rational framework in which sound decisions could be made.

8.3 Lessons Learned

Though the application methodology is valid, it is nonetheless a complex process due to the consistent value assessments of probability and utility that must be provided by the expert.

In order to develop a database which could realistically manipulate several different classes of problems would be very time consuming, not to mention extremely hard to control. Though the detail needed by any large-scale, rule-based expert system would be extensive, a system founded on this type of reasoning scheme would require a more rigorous validation process due to the functional nature of the rules.

Though the assumptions of conditional independence of evidences and mutual exclusiveness of hypotheses help with computations, they nonetheless limit potential areas of application. The restrictive nature of the single recommended actions might also be unrealistic in some real world applications. Nevertheless, as stated at the onset of the thesis, this type of knowledge base reasoning is more suited towards problems that lend themselves to probabilistic representation and involve uncertainty reasoning.

8.4 Sensitivity of Belief Assessments

In this scenario, decision theory proved to be a viable expert system reasoning methodology, however the successful application of this inference method was nonetheless highly dependent on the expert's assessment of the probabilities and utilities. Had these values been different, the results could have been less successful.

Though I agree that probability is the only logical representation for uncertainty in expert systems, there is a serious question concerning its application and assessment in real world decision making. This is not referring to its manipulation (this thesis showed how that can be done), but rather how and to what situations it can be applied.

For example, if I develop an expert system's data base with my subjective opinions and beliefs, the system is biased towards my assessments which may or may not correlate with another expert's or a particular user's. This bias is woven into the likelihood and utility data and directly effects the system output. Consequently, problems could arise in situations in which my belief in an evidence value

is .9 and yet a user's belief is .7. Given the disparity, a different course of action could be pursued which, given the scenario under which the system was developed, was inadvertantly recommended. Therefore, a reasoning system of this sort must also provide restrictions as to under what situations a user can question the veracity of the information. Otherwise, though the system recommended action will be rational given the data it is working with, it may or may not coincide with what the expert would have done in the same situation. With a system of this sort, added responsibility comes with the added versatility.

8.5 Summary

This thesis demonstrated that a decision-theoretic approach towards rule-based expert systems offers both strength in theory and application. With satellite anomaly resolution as a model, a prototype decision-analytic reasoner was developed which manipulated uncertainty in a sound probabilistic manner and combined it with utility to rationally recommend courses of action. Though it is by no means close to being a functional expert system, it nonetheless demonstrates the underlying concepts that make decision analysis both a logical and rational choice for further development in expert system reasoning schemes.

8.6 Recommendations

Though decision theory is unquestionably a valid reasoning methodology for expert system development, this thesis raised a few questions which need to be answered in order to further determine the validity and useability of this approach in larger more complex problem domains. The following are recommendations which can help achieve this goal:

1. Expand the DA reasoner to include nested hypotheses. This would allow the system to still work with small evidence sets yet would permit much larger problems to be handled.

2. Explore new and more efficient ways of determining probabilities. This could involve probabilistic knowledge acquisition tools such as belief networks or influence diagrams. This would further insure that knowledge obtained from the expert would be more representative of his actual beliefs.

3. Determine a quick method in which a user can determine exactly which values have the most effect on the output (i.e., priors, likelihoods, or utilities). This can help in fine tuning the system in order to assist with development and validation.

4. Combine a decision theoretic rule-base with a model-based expert system. A hybrid system like this would offer much more than either a rule-based or model-based system could alone.

Appendix A. *GPS Subsystems*

The following paragraphs give a summary of each of the GPS satellite subsystems.

A.1 *Structure Subsystem*

The structure subsystem consists of the following:

1. Primary and secondary structure
2. Mechanisms
   a. Solar array deployment
   b. Forward TT&C mast deployment
   c. Passive nutation damper release.

The primary and secondary structure are completely passive and neither receive commands nor produce telemetry outputs. The solar arrays are folded along the SV sides during launch and initial orbital operations and are deployed when configuring the vehicle for three-axis stablization. If the vehicle is launched from the shuttle, the forward TT&C mast deployment is provided by the PAM-D II after separation from the orbiter. The passive nutation damper is released by ground control command shortly after first acquistion.

A.2 *Thermal Control Subsystem*

The Thermal Control Subsystem (TCS) provides space vehicle and equipment temperature control within accepted limits during all mission phases. Passive thermal control is accomplished by thermal coatings, insulation blankets, thermal shields and heat transfer doublers. Active thermal control is provided by heaters, thermal switches, and frequency standard active baseplate temperature control units (ABTCU). Heat rejection is provided by semi-active louvers and direct radiators which are maintained parallel to the sun line by the AVCS and by passive battery radiators. Minimum temperatures for the EPS batteries, OIS, LBS, and the wetted RCS component are maintained by heaters and thermostat controls.

A.3 *Electrical Power Subsystem*

ne Electrical Power Subsystem (EPS) provides for the generation, storage, control, and distribution of electrical power to all space vehicle subsystems. The EPS is capable of supplying all required power for steady-state and transient loads.

The solar arrays provide primary electrical power during all phases of mission operations following sun acquisition. Three nickel-cadmium batteries are also charged by the solar arrays to provide power during eclipses, and during the launch and orbit injection phases when insufficient power is available from the stowed arrays. In addition to the solar arrays and batteries, the EPS includes power conditioning equipment for the bus power conditioning and battery charging, a load control unit for controlling and distributing power to the loads, a solar array drive and power transfer assembly for positioning the solar arrays and transferring the power to the main bus, and a DC/DC converter for providing secondary DC voltages.

A.4 *Attitude and Velocity Control Subsystem*

The Attitude and Velocity Control subsystem (AVCS), in conjuction with the other SV subsystems, provides for space vehicle (SV) attitude control and velocity corrections and control subsequent to separation from the PAM-D II to satisfy the operational, thermal, electrical power, and payload and subsystem pointing requirements. The AVCS also provides active nutation damping prior to SV separation from the PAM-D II and passive nutation damping during subsequent periods of spin-stabilized operations.

The SV has two stablization modes—an inertially stablized spin mode and a three-axis stablized mode with the SV +Z axis aligned along the earth nadir. The spin-stablizied mode is capable of SV/PAM-D II or ELV launch separation stabliziation, long-term on-orbit storage, attitude sensing for attitude determination, precession maneuvers, and large (5-lbf) thruster drift orbit corrections. The three-axis stabilized mode provides all on-station attitude control, including the capability to acquire the earth and sun, provide payload and space vehicle pointing, perform unrestricted small (0.1-lbf) orbit corrections, and stationkeeping.

Attitude control is accomplished either autonomously or from the ground with override of autonomous functions. Velocity maneuvers are ground controlled and support drift orbit injections correction, final orbit insertion, orbit phasing, and orbit period adjustment.

The three-axis stabilized space vehicle maintains earth nadir and solar array pointing through a combination of yaw steering and solar array orientation. The single degree-of-freedom solar array drives about the space vehicle (SV) Y-axis (pitch axis) and the space vehicle yaw steers about the Z-axis (local vertical). The space vehicle yaws in orbit to maintain the sun in the space X-Y plane by using the yaw sun sensor located on the solar arrays. Nadir sensing is accomplished with a static earth sensor. The solar array is maintained normal to the sun line with the pitch sun sensor mounted on the solar arrays.

A.5 *Orbit Insertion Subsystem*

The Orbit Insertion subsystem (OIS) consists of a single solid rocket motor which provides the required impulse to boost the space vehicle from the transfer orbit into an initial drift orbit. OIS ignition is controlled by ground command and based on mission planning activities supported by software resident at the Air Force Mission Control Center (MCC).

A.6 *Reaction Control Subsystem*

The Reaction Control subsystem (RCS) provides the necessary impulse to perform SV translation and spin vector precession maneuvers during the SV/PAM-D II transfer orbit operations, SV drift orbit operations, and SV on-orbit operations. The RCS is a blowdown system utilizing monopropellant hydrazine pressurized by gaseous nitrogen. The subsystem consists of a propellant/pressurant storage (PPS) assembly, a propellant distribution and control (PDC) assembly, and the rocket engine assembly module (REAM).

The PPS components consist of two propellant tanks, each equipped with a positive expulsion diaphram, two propellant fill and drain valves, two pressurant fill valves, and two temperature transducers.

The PDC components consist of two pressure transducers, two filters, and two latching isolation valves.

The REAM components consist of twenty 0.44-N (0.1-lbf) thrusters and two 22.2-N (5-lbf) thrusters, each equipped with a temperature sensor, thruster valve heaters, and catalyst bed heaters. The 5-pound thrusters, in conjuction with the AVCS, are used to perform SV translation

and spin vector precession maneuvers during spin-stabilized operations. The 0.1-pound thrusters are used to perform translation (stationkeeping and rephasing) maneuvers and provide backup SV attitude control during three-axis stabilized operations.

A.7 Telemetry, Tracking and Commanding Subsystem

The Telemetry, Tracking and Commanding (TT&C) subsystem performs telemetry, tracking and commanding functions, as required, to support space vehicle and launch vehicle (PAM-D II/Orbiter) operations; and is compatible with the Air Force Satellite Control Facility (AFSCF), the Space Ground Link System (SGLS) and the Operational Control System. The TT&C subsystem supports the integration, checkout and launch of the SV in the PAM-D II/Orbiter launch vehicle and supports the upload of processor data, RFDU, NDU and DCEA, via the TT&C receiver/demodulators and signal conditioner from the OCS. The TT&C subsystem receives commands via the uplink from the OCS RTSs and processes this data for control of SV subsystems.

The TT&C subsystem also collects, processes and transmits SV telemetry data including Navigation Data Unit (NDU) memory dump data and is capable of supporting non-coherent pseudo-random noise (PRN) turnaround ranging.

A.8 Navigation Payload

The Navigation Payload (NPD) consists of four Frequency Standard Assemblies (FSA), the Frequency Synthesizer and Distribution Unit (FSDU), and the Navigation Data Unit (NDU). The Navigation Payload, in its primary capacity, generates pseudo-random noise codes and modulo-two adds navigation data to them prior to their output to the L-band subsystem where they modulate the L-band carrier frequencies.

A.9 L-Band Subsystem

The L-band subsystem (LBS) consists of the devices required to generate, amplify, filter, combine, and transmit signals at L1, L2, and L3 carrier frequencies. Interface devices from acceptance of uplink commands from, and exchange of data with the TT&C subsystem are included for control, update, and function monitoring of the L-band subsystem.

A.10 Nuclear Detonation Detection System

The Nuclear Detonation (NUDET) Detection System (NDS) consists of the Integrated Transfer Subsystem (ITS) and the Global Burst Detector (GBD).

A.10.1 Integrated Transfer Subsystem (ITS). The Integrated Transfer Subsystem (ITS) provides the NUDET Detection System (NDS) with a satellite-to-satellite UHF communication link. The ITS provides a data relay to other space vehicles by means of a time-division, multiple-access (TDMA) UHF crosslink data. The TDMA mechanization provides a UHF crosslink data 1.5 second time slot for each ITS equipped space vehicle and accomodates up to 24 satellites.

A.10.2 Global Burst Detector (GBD). The Global Burst Detector (GBD) consists of the processor and those sensors used in the detection of nuclear detonations. The following components constitute the GBD:

1. Burst Detector Processor (BDP)

2. Burst Detector Optical Sensor (BDY)

3. Burst Detector X-Ray Sensor (BDX) or

4. Burst Detector Dosimeter (BDD)- SV's 18, 24, 28, 33, and 39.

A.10.3 GPS Cargo Element Should the satellite be launched from the shuttle, the SV itself is a subsystem of the launch vehicle. The SV, PAM-D II, and cradle assembly create a cargo element in the space shuttle's cargo bay.

This concludes a brief summary of the GPS subsystems.

Appendix B. *Program Outline*

The scheme program TEST3.S was written in order to demonstrate the applicability of uncertainty management and utility theory in expert system development. The program is approximately 600 lines long and serves as a prototype inference engine of a decision analytic reasoner. The program accesses a separately loaded file which serves as the knowledge base of the particular problem domain. This knowledge base contains the priors, the joint likelihood table, and the utility action-matrix. The priors are the user provided values which are written in a scheme construct called "priors". This is how the prior values .6, .2, .1, and .1 would be represented.

```
(define (priors)
'(.6 .2 .1 .1) )
```

The joint likelihood table is a Quattro^{TM} generated file which has been altered in order to fit into scheme syntax. This data file is labeled "likelihoods". Below is a hypothetical example.

```
(define (likelihoods)
 '( ((() (1 2 3)) (.1  .1   .1    .1))
    (((1)  (2 3)) (.2  .6   .3    .4))
          .                .
          .                .
          .                .
    (((1 2 3) ()) (.02 .024 .042 .024)) ))
```

The action-matrix defines the utility values which the expert has determined for each action. The list is constructed with the different actions representing the different rows and the possible hypotheses of the problem domain representing the

different columns. Below is how the action matrix is represented in the knowledge base.

	H1	H2	H3	H4
A1	100	10	20	15
A2	5	100	6	30
A3	70	80	100	10
A4	10	20	30	100

```
(define (action-matrix)
  '( (100 10 20 15)
     (5 100 6 30)
     (70 80 100 10)
     (10 20 30 100) ))
```

After loading TEST3.S and the appropriate file containing the priors, likelihoods, and action-matrix values, the system is ready for use. Below is the step-by-step process for using the program.

Step 1. *type:* pcs

This loads scheme program into the computers resident memory. (This assumes the scheme program is resident on the default drive.)

Step 2. *type:* (load "a:test3.fsl")

This loads the fastload version of the TEST3.S from the floppy drive into the scheme buffer.

Step 3. *type:* (load "a:knowledgebase")

This loads the user created knowledge base. This file name is whatever the user has named it.

Step 4. *type:* (start)

This starts the program, asking the user to indicate what evidence he has observed. First, enter the evidence values that were observed as true and the belief in that evidence. Then enter the evidences that were observed as false and their corresponding belief value.

```
        T             F
(((1 .3)(2 .1))((4 .1)(7 .8)))
```

In the above example, Evidence 1 and 2 were observed as true and 4 and 7 as false. With a belief in Evidence 1 of .3 and a belief in Evidence 2 of .1. The associated beliefs in Evidences 4 and 7 are .1 and .8, respectively. The 'true' set consists of the subsets (1 .3) and (2 .1), while the false subsets are (4 .1) and (7 .8). Please note that there must be a space in between the evidence and the belief value. The belief values can range from 0 to 1.

After the values are entered, hit the return key. The program will now take the entered values and, utilizing a combination of Bayes' Theorem and decision theory, calculate the appropriate utility values for each action. The action yielding the highest utility will be recommended by the system as the rational course of action. A normalized list will also output the percentage of utility associated with each action. Action values are vertically output in ascending order starting from the top.

A1	10
A2	30
A3	40
A4	20

Do action 3

In order to validate the accuracy of the output data, several data runs were accomplished testing the approximately 35 subroutines that are accessed during the programs operation.

The program's calculations can be broken up into 6 major steps. Following is a short description of each step. Step 1 breaks the entered evidences into all possible subset combinations.

```
(i.e., (1 2 3) -> (()(1 2 3))
                  ((1) (2 3))
                  ((2) (1 3))
                  ((3) (1 2))
                  ((1 2) (3))
                  ((1 3) (2))
                  ((2 3) (1))
                  ((1 2 3)()) ).
```

Step 2 takes each component list and retrieves the associated likelihood values from the joint likelihood matrix.

```
(i.e., (1 2 3) -> (()(1 2 3)) (.1  .1   .1   .1)
                  ((1) (2 3)) (.2  .6   .3   .4)
                  ((2) (1 3)) (.5  .1   .2   .2)
                  ((3) (1 2)) (.2  .4   .7   .3)
                  ((1 2) (3)) (.1  .06  .06  .08)
                  ((1 3) (2)) (.04 .24  .21  .12)
                  ((2 3) (1)) (.1  .04  .14  .06)
                  ((1 2 3)()) (.02 .024 .042 .024).
```

Step 3 takes each component list and calculates the corresponding belief vector. This vector is determined by the placement of the evidences in the sublist and their original placement as entered by the user. For example, if the values are in the same category (i.e., true or false) as entered by the user, the entered belief values are used.

If the values are in opposite list, the entered belief value substracted from 1 is used. For example if the entered list is

```
                                                 T          F
<original user entered list>   (((1 .3)(2 .6))((3 .8)))

<Example                    T    F                   resultant vector
 component sublist>    ((1 2)  (3))  (.3)(.6)(.8)       = (.3)(.6)(.8)
                       ((2 3)  (1))  (.6)(1-.8)(1-.3) = (.6)(.2)(.7)
```

These belief vectors are multiplied by the likelihood values in order to give their respective weights.

```
<Example                              H1    H2   H3    H4
 component sublist>    ((1 2)  (3))  (.1    .06  .06   .08) * (.144)
                       ((2 3)  (1))  (.1    .04  .14   .06) * (.084)
```

```
The resultant likelihood values are:

                            H1      H2       H3       H4
          ((1 2)  (3))  (.0144  .00864   .00864   .01152)
          ((2 3)  (1))  (.0084  .00336   .01176   .00504)
```

Step 4 adds all of the likelihood values together to get the total scaled likelihood associated with each hypothesis. Though I am only showing two likelihoods, this value would include all subset lists of the entered evidences.

```
                            H1      H2       H3       H4
          ((1 2)  (3))  (.0144  .00864   .00864   .01152)
          ((2 3)  (1))  (.0084  .00336   .01176   .00504)
                        -------------------------------
                          .0228  .0348    .0204    .01656
```

Step 5 multiplies each of the hypothesis likelihood values by its respective prior in order to determine the posterior value (assume priors are (.6 .2 .1 .1)).

```
                  H1           H2           H3           H4
(1 2) (3) (.0228)(.6) (.0348)(.2) (.0204)(.1) (.01656)(.1)

(1 2) (3)   (.01368)     (.00696)     (.00204)     (.001656)
```

Step 6 takes the resultant posteriors and does a dot product with each row of the action-matrix in order to determine the utility values.

```
[.01368 .00696 .00204  .001656] * | 100 10   20   15 | = 1.50324
                                  | 5   100  6    30 | =  .82632
                                  | 70  80   100  10 | = 1.73496
                                  | 10  20   30   100! =  .5028
```

The largest value is selected and the associated action is recommended. In this case it is Action 3. The values are also normalized and the relative percentage of each utility value is output.

Presently as it stands, the hypotheses and actions are limited to 4 values while there is no limit on the number of evidences which can be used in the knowledge base. This thesis used 7 evidences which meant during each run, 128 different sublists were manipulated. As the number of evidences increases, the time determining the output will vary due to the factor of two that is added for each evidence.

Appendix C. *Scheme Code*

```
;;---------------------------------------------------------
;; Decision-Analytic Code. The language is SCHEME
;; Written by Grady Elliott for Thesis work. 1 - 8 Oct 1989
;;----------------------------------------------------------

;;------------------------------------------------------
;; PRIORS
;; These are the prior values of the hypothesis set. They
;; should be put into the particular problem data set.
;;------------------------------------------------------

;(define (priors)
;   '(.6 .2 .1 .1))

;;-----------------------------------------------------------
;; LIKELIHOODS
;; This shows the form in which the likelihood values from
;; the p{e}|h need to entered in. The likelihoods show be
;; put into the particular problem data set. (These have been
;; commented out.)
;;---------------------------------------------------------

;(define (likelihoods)
;  '(  ((() (1 2 3)) (.1  .1   .1   .1))
;      (((1)  (2 3)) (.2  .6   .3   .4))
;      (((2)  (1 3)) (.5  .1   .2   .2))
;      (((3)  (1 2)) (.2  .4   .7   .3))
;      (((1 2)  (3)) (.1  .06  .06  .08))
;      (((1 3)  (2)) (.04 .24  .21  .12))
;      (((2 3)  (1)) (.1  .04  .14  .06))
;      (((1 2 3) ()) (.02 .024 .042 .024)) ))

(define (values)
  '(((1 .9)(2 .8))((3 .7)(4 .6))) )

;;-------------------------------------------------------------
;; ACTION-MATRIX
;; This is the action utility diagram. Each row is for a
;; particular action, 1 - 4. Use particular matrix for each
;; problem data set. (Example below has been commented out)
;;-------------------------------------------------------------
;(define (action-matrix)
;  '((100 10 20 15)
;    (5 100 6 30)
;    (70 80 100 10)
;    (10 20 30 100)))
```

```
;;-------------------------------------------------------------
;; ACTION
;; This procedure recommends 1 of 4 actions depending upon
;; the utility values of the results.
;;-------------------------------------------------------------

(define (action lst)
   (cond ((null? lst) '())
         ((equal? (car lst) (car (reverse (sort lst))))
           (display "Do action 1"))
         ((equal? (cadr lst) (car (reverse (sort lst))))
           (display "Do action 2"))
         ((equal? (caddr lst) (car (reverse (sort lst))))
           (display "Do action 3"))
    (else
      (display "Do action 4"))))

;;-----------------------------------------------------------
;; ADD
;; This procedure adds values in a vector to obtain the total
;; resultant values. Used when all evidence is not observed.
;;---------------------------------------------------------

(define (add lst1 lst2)
   (cond ((null? lst2) lst1)
     (else
       (append(list (+ (car lst1) (car lst2)))
              (add (cdr lst1) (cdr lst2))))))


;;-----------------------------------------------
;; START   (BGN)
;; This procedure starts the whole program.
;; Ask user to input the evidences he observed and the
;; belief in that evidence.
;;-----------------------------------------------------

(define (start)
  (bgn)
  (reset))

(define (bgn)
  (window-clear 'console)
  (window-delete *percent-window*)
  (display "Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6))) ")
  (newline)
  (newline)
  (let* ((input1 (read))
         (input2 input1))
         (myprop input2)
```

```
  (total
   (posterior
     (bigadd
      (big-build
        (d-total(do-all(flat(extract input1))) (likelihoods))
          (extract input1))) (priors)) (action-matrix)) ))


;;-------------------------------------------------
;; BIG-ADD
;; This procedure is used to compute total values of the
;; likelihood vectors after the BIG-BUILD routine. It takes
;; one list of many vectors and outputs a single vector
;; with the sum of all the sublist vectors.
;; [263] (bigadd '((1 2 3) (1 2 3) (1 2 3)))
;;  = > (3 6 9)
;;----------------------------------------------------

(define (bigadd lst)
  (cond ((null? lst) '())
    (else
      (add (car lst) (bigadd (cdr lst))))))


;;-------------------------------------------------------
;; BIG-BUILD
;; This procedure is used on the output of D-TOTAL. Its
;; purpose is to output a list of vectors which have
;; been adjusted to the value determined by the DETERMINE and
;; MULT procedures. It accesses the SCALAR routine and BIG2
;;----------------------------------  ----------------------

(define (big-build lst goodlst)
   (cond ((null? lst) '())
   (else
     (append
       (list (scalar (cadar lst) (big2 (caar lst) goodlst)))
       (big-build (cdr lst) goodlst))))))

(define (big2 lst1 lst2)
    (cond ((null? lst1) '())
     (else
      (mult (determine lst1 lst2))))))
```

```
;;------------------------------------------------------------
;; BUILD
;; This procedure builds the double list needed by joining the
;; incoming list, with a list of the reverse of the members in
;; in each li... ( ((1) (2 3)) ((3) (5 6)) + ((2 3)(1)) (5 6)
;; (3)) )
;;-----  ---------------------------------------------------

(define (build lst)  ;builds double the list reversing the order
  (cond ((null? lst) '())
     (else
      (append lst (flip lst)))))

;;-------------------------------------------------------
;; CHECK
;; (old) This procedure was used when I initially didn't take
;; uncertainty of evidence into consideration. It adds the
;; vector values if the evidence meets criteria.
;;------------------------------------------------------

(define (check lst1 lst2)
   (cond ((null? lst2) '())
      ((subset? lst1 (caaar lst2)) (add (cadar lst2) (check lst1 (cdr lst2))))
    (else
      (check lst1 (cdr lst2)))))

;;----------------------------------------------------------
;; CLIP
;;This procedure builds sets of head to tail throughout the list
;; (1 2 3 4 5 6) => (1 6) (2 5) (3 4)
;;----------------------------------------------------------

(define (clip lst)
  (cond ((null? lst) '())
   (else
    (append
     (list
     (append (list (car lst)) (list (car (reverse lst)))))
      (clip (remove (cdr lst))))))


;;------------------------------------------------------
;; D-CHECK
;; This procedure used to check to if I can build proper
;; subset list (i.e. (evidences + (summed likelihoods))
;; car of the do-all list and likelihood list. It accesses
;; the NEWCHECK procedure.
;;-----------------------------------------------------

(define (d-check lst1 lst2)
  (append (list lst1) (list (newcheck lst1 lst2)) ))
```

```
;;-----------------------------------------------------
;; D-TOTAL
;; This procedure produces the hugh list of the different
;; combinations with the sum total of the likelihood values
;; generated in the NEWCHECK procedure.
;;--------------------------------------------------------

(define (d-total lst1 lst2)
   (cond ((null? lst1) '())
     (else
    (append
      (list (d-check (car lst1) lst2)) (d-total (cdr lst1)
      lst2)))))

;;-------------------------------------------------
;; DELETE
;; This procedure deletes a member from a list.
;;-------------------------------------------------

(define (delete x lst)
   (cond ((null? lst) '())
         ((equal? x (car lst)) (delete x (cdr lst)))
      (else
         (cons (car lst) (delete x (cdr lst)) ))))

;;--------------------------------------------------------
;; DETERMINE
;; This procedure checks to see if the placement of the
;; values of the list in question, matches that of the list
;; created by the user with the read statement. It determines
;; the values of the evidence that are going to be used to
;; multiply by the posterior values. These values are retrieved
;; from the property list which has assigned the values
;; depending upon the user read input. If placement doesn't
;; match, then one minus the value is given. A list of the
;; evidence vector is output. DETERMINE2 is consulted.
;;-------------------------------------------------------

(define (determine lst goodlst)
  (append (determine2 (car lst) (car goodlst))
          (determine2 (cadr lst) (cadr goodlst))))

(define (determine2 lst goodlst)
   (cond ((null? lst) '())
         ((null? (car lst)) '())
         ((member (car lst) goodlst)
           (cons (getprop 'grady (car lst))
                 (determine2 (cdr lst) goodlst)))
      (else
         (cons (- 1 (getprop 'grady (car lst)))
               (determine2  (cdr lst) goodlst))) ))
```

```
;;--------------------------------------------------------------
;; DO-ALL combines the output of the BUILD, CLIP, and SUBSETS
;; routines. It gives a list broken down into all of its subsets
;;--------------------------------------------------------------

(define (do-all lst)
  (cond ((null? lst) '())
     (else
        (build (clip (subsets lst))))))

;;---------------------------------------------------------
;; DOT
;; This procedure figures the dotproduct of two one-row vectors
;;---------------------------------------------------------

(define (dot lst1 lst2)
    (cond ((null? lst1) 0)
          ((null? lst2) 0)
        (else
         (+ (* (car lst1) (car lst2)) (dot (cdr lst1) (cdr lst2))))))

;;-----------------------------------------------------------
;; DOTP
;; This procedure figures the dot product of a 1 one-row vector
;; and a multirow matrix. Produces a matrix list.
;;-----------------------------------------------------------

(define (dotp lst1 lst2)
    (cond ((null? lst1) '())
          ((null? lst2) '())
     (else
       (append (list (dot lst1  (car lst2))) (dotp lst1 (cdr lst2))))))

;;-------------------------------------------------------
;; EXTRACT
;; This procedure is used to generate the list of evidences
;; without the subjective belief assessments.
;; ( ((1 .9) (2 .8)) ((3 .4) (4 .7))) => ((1 2) (3 4))
;; It accesses the EXTRACT2 procedure.
;;-----------------------------------------------------------

(define (extract lst)
  (append (list (extract2 (car lst)))
          (list (extract2 (cadr lst))) ))

(define (extract2 lst)
   (cond ((null? lst) '())
     (else
       (cons (caar lst)
             (extract2 (cdr lst)) ))))
```

```
;;---------------------------------------------------
;; FLAT
;; This procedure links the FLATTEN and FLATTEN2 procedures
;; together.
;;---------------------------------------------------

(define (flat lst)
   (flatten2 (flatten lst)))

;;---------------------------------------------------
;; FLATTEN
;; This procedure flattens out a list of many sublist into
;; one major list. ((1)(2)(3)) => (1 2 3)
;;---------------------------------------------------

(define (flatten lst)
   (cond ((null? lst) '())
         ((atom? (car lst))
          (append (list (car lst)) (flatten (cdr lst))))
       (else
         (append (flatten (car lst)) (flatten (cdr lst))) )))

;;---------------------------------------------------
;; FLATTEN2
;; This procedure is used to alleviate problems which arise
;; when the flattened list has nil as a member. (i.e.
;; '(1 2 ()). Needs to be done before the DO-ALL procedure.
;; This is for cases when either no evidence was true or no
;; evidence was false.
;;---------------------------------------------------

(define (flatten2 lst)
   (cond ((null? lst) '())
         ((member '() lst) (delete '() lst))
      (else
         lst)))

;;---------------------------------------------------
;; FLIP
;; This procedure reverses the members inside of a list.
;; ((1 2) (3)) = > ((3) (1 2))
;;---------------------------------------------------

(define (flip set)
  (cond ((null? set) '())
    (else
      (append
         (list (reverse (car set))) (flip (cdr set))))))
```

```
;;-------------------------------------------
;; LARGEST
;; This procedure finds the largest number in a list
;; hopefully I will be able to use this in the ACTION
;; routine.
;;---------------------------------------------

(define (largest lst)
   (cond ((null? lst) '())
         ((null? (cdr lst)) (car lst))
         ((> (car lst) (cadr lst)) (largest (cons(car lst) (cddr
         lst))))
      (else
         (largest (cdr lst)))))

.;---------------------------------------------
;; MULT
;; This procedure takes the members of a vector and multiplies
;; them by one another to get the SCALAR value.
;; [3] (mult '(2 3 4)) => 24
;;---------------------------------------------

(define (mult lst)
  (cond ((null? lst) 1)
     (else
       (* (car lst) (mult (cdr lst))))))

;;---------------------------------------------------
;; MYPROP
;; This builds a property list which keeps track of the
;; belief of each of the evidences. It accesses MYPROP2
;; [39] (myprop '( ((1 .9) (2 .8)) ((3 .7) (4 .2)) ) )
;;  returns =>   (0.9 0.8 (4 0.2))
;; [42] (getprop 'grady 3)
;;   0.7
;;   [43] (getprop 'grady 1)
;;   0.9
;;   [44] (getprop 'grady 2)
;;   0.8
;;   [45] (getprop 'grady 4)
;;   0.2
;;-----------------------------------------------------

(define (myprop lst)
   (append (myprop2 (car lst)) (myprop2 (cadr lst))))

(define (myprop2 lst)
   (cond ((null? lst) '())
     (else
       (cons (putprop 'grady (cadar lst) (caar lst))
             (myprop2 (cdr lst))))))
```

```
;;------------------------------------------------------
;; NEWCHECK
;; This procedure adds vector values if the evidence meets
;; the correct position criteria. LST2 is LIKELIHOODS values
;;---------------------------------------------------------

(define (newcheck lst1 lst2)
  (cond ((null? lst2) '())
   ((and (subset? (car lst1) (caaar lst2)) (subset? (cadr lst1) (cadaar lst2)))
(add (cadar lst2) (newcheck lst1 (cdr lst2))))
    (else
     (newcheck lst1 (cdr lst2)))))

;;-----------------------------------------------------
;; PERCENT
;; This procedure is used to tell the percentage each
;; member makes up of the list. It accesses the PERCENT2
;; and SUMLIST procedures.
;; i.e. (percent '(2 2 2 2)) => (.25 .25 .25 .25)
;;----------------------------------------------------

(define (percent lst)
  (let ((total (sumlist lst)))
        (percent2 lst total)))

(define (percent2 lst lst2)
   (cond ((null? lst) '())
      (else
        (cons ( / (car lst) lst2)
              (percent2 (cdr lst) lst2)))))

;;-----------------------------------------------------
;; POSTERIOR
;; This procedure takes the output from BIGADD and figures
;; out the posterior values of each of the hypotheses. This
;; is the vector that will be used in the calculation of
;; the utility values through the action matrix.
;;----------------------------------------------------

(define (posterior lst1 lst2)
   (cond ((null? lst1) '())
         ((null? lst2) '())
     (else
        (append (list (* (car lst1) (car lst2))) (posterior (cdr
        lst1) (cdr lst2))))))
```

```
;;------------------------------------------------------
;;PREFIX
;; This procedure is used to help fathom all the subsets of
;; a particular list
;;-----------------------------------------------------

(define (prefix elt lst) ;a first element to help expand set
   (cond ((null? lst) '())
     (else
       (cons (cons elt (car lst))
               (prefix elt (cdr lst)) ))))

;;--------------------------------------------------------
;; REMOVE
;; This procedure takes the tail off of a list. This routine is
;; accessed by the CLIP to build the needed lists.
;;--------------------------------------------------------

(define (remove lst)
  (cond ((null? lst) '())
     (else
      (reverse (cdr (reverse lst))) )))

;;----------------------------------------------------
;; SCALAR
;; This procedure is used to take the multiplication number
;; given by the DETERMINE and MULT and multiply it across the
;; the vector given the NEWCHECK procedure.
;; [105] (scalar '(1 2 3 4 5 6) 100)
;; (100 200 300 400 500 600)
;; Then this vector will be added with all of the SUBSETS vectors
;; which will finally be used in the POSTERIOR routine.
;;-------------------------------------------------------

(define (scalar lst x)
   (cond ((null? lst) '())
      (else
        (cons (* (car lst) x) (scalar (cdr lst) x)))))

;;------------------------------------------------------
;; SORT
;; This procedure sorts a list from lowest to highest values.
;; It accesses the INSERT procedure which places the sorted
;; values.
;;-----------------------------------------------------

(define (sort lst)
  (cond ((null? lst) '())
    (else
      (insert (car lst) (sort (cdr lst))) )))
```

```
(define (insert x sorted-list)
   (cond ((null? sorted-list) (list x))
         ((< x (car sorted-list)) (cons x sorted-list))
      (else
        (cons (car sorted-list) (insert x (cdr sorted-list))))))

;;-----------------------------------------------------------
;; SUBSET?
;; This procedure determines if sets are subsets of each other
;;---------------------------------------------------------

(define (subset? lst1 lst2)
   (or (null? lst1)
       (and (member (car lst1) lst2)
            (subset? (cdr lst1) lst2))))

;;-----------------------------------------------------------
;; SUBSETS
;; This procedure finds all the subsets of a list.
;; It access the prefix routine.
;; (subsets '(1 2 3))
;; (() (3) (2) (2 3) (1) (1 3) (1 2) (1 2 3))
;;-----------------------------------------------------------

(define (subsets set) ;finds all subsets of a set of values
  (cond ((null? set) (list nil))
    (else
      (append (subsets (cdr set))
              (prefix (car set)
                      (subsets (cdr set)) )))))

;;-----------------------------------------------------------
;; SUMLIST
;; This procedure adds the members of the list to give the
;; total value. It is used in the PERCENT procedures.
;; (sumlist '(1 2 3 4)) => 10
;;-----------------------------------------------------------

(define (sumlist lst)
   (cond ((null? lst) 0)
      (else
         (+ (car lst) (sumlist (cdr lst))))))
```

```
;;--------------------------------------------------------
;; TOTAL
;; This procedure takes the dot product of hypothesis set and
;; ACTION-MATRIX values and outputs action as determined in the
;; ACTION routine.
;;--------------------------------------------------------

(define (total lst1 lst2)
   (cond ((null? lst1) '())
         ((null? lst2) '())
    (else
    (vertprint (dotp lst1 lst2))

    (window-set-size! *percent-window* 4 22)
    (window-set-position! *percent-window* 5 22)
    (window-clear *percent-window*)
    (set-fluid! output-port *percent-window*)
    (vertprint (percent (dotp lst1 lst2)))
    (set-fluid! output-port 'console)
    (newline) (newline)
    (action (dotp lst1 lst2)))))

(define *percent-window* (make-window #f #t))

;;------------------------------------------------------
;; VERTPRINT
;; This procedure vertically prints out a list.
;;------------------------------------------------------

(define (vertprint lst)
   (cond ((null? lst) *the-non-printing-object*)
     (else
       (print (car lst))
       (vertprint (cdr lst)))))

;;------------------------------------------------------
;; The following procedures were used as test procedures to
;; check and validate the code.
;;------------------------------------------------------

(define (small-action)
 '((1 2 3 4)
   (2 3 4 1)
   (2 2 2 2)
   (1 1 1 1)))


(define (test)  ; test routine to determine if total routine worked
   (print "Enter 4 values of the hypothesis set") ; with user input read
    (total (read) (action-matrix))) ; statement.
```

```
;;------------------------------------------------------
;; GROUPLIST
;; This is a practice hypothesis/likelihood table to check
;; numbers.
;;-----------------------------------------------------

(define (grouplist)
 '( (((1) (2 3)) (1 2 3 4))
    (((2) (1 2)) (1 2 3 1))
    (((1 2) (3)) (1 1 1 1)) ))

;;------------------------------------------------------
;; SMALL-ACTION
;; This is a practice action table to check numbers.
;;------------------------------------------------------

(define (small-action)
 '((1 2 3 4)
   (2 3 4 1)
   (2 2 2 2)
   (1 1 1 1)))
```

Appendix D. *GPS Knowledge Base*

The following listing is the knowledge base accessed by the DA reasoner (Test3.s). The values used for the priors, likelihoods, and utilities are those provided by the user, written in scheme syntax. The Quattro^{TM}-generated file was used as the source for the likelihood values.

```
(define (priors)
  '( 9766  0.001953  0.001953  0.01953) )

(define (action-matrix)
  '( (100    200     1000    500)
     (0      10000   0       500)
     (40     1000    2000    3000)
     (10     4000    200     5000) ))

(define (likelihoods)
  '(
   ((()(1 2 3 4 5 6 7)) (1.04e-5    6.35e-8    1.1e-6     7.79e-8))
   (((1) (2 3 4 5 6 7)) (0.010376   6.34e-5    1.1e-9     7.79e-5))
   (((2) (1 3 4 5 6 7)) (5.22e-8    3.19e-10   9.87e-05   3.92e-10))
   (((3) (1 2 4 5 6 7)) (1.04e-8    6.35e-11   0.001096   7.8e-11))
   (((4) (1 2 3 5 6 7)) (9.35e-5    6.28e-6    9.88e-6    7.72e-6))
   (((5) (1 2 3 4 6 7)) (4.43e-5    7.3e-7     1.1e-10    1.95e-8))
   (((6) (1 2 3 4 5 7)) (1.04e-9    2.71e-7    1.1e-10    1.59e-9))
   (((7) (1 2 3 4 5 6)) (8.5e-6     8.76e-8    1.1e-9     7.72e-6))
   (((1 2) (3 4 5 6 7)) (5.21e-5    3.19e-7    9.88e-8    3.91e-7))
   (((1 3) (2 4 5 6 7)) (1.04e-5    6.35e-8    1.1e-6     7.79e-8))
   (((1 4) (2 3 5 6 7)) (0.093383   0.006276   9.89e-9    0.007707))
   (((1 5) (2 3 4 6 7)) (0.044234   0.000729   1.1e-13    1.95e-05))
   (((1 6) (2 3 4 5 7)) (1.04e-06   0.00027    1.1e-13    1.59e-06))
   (((1 7) (2 3 4 5 6)) (0.008489   8.75e-05   1.1e-12    0.007707))
   (((2 3) (1 4 5 6 7)) (5.22e-11   3.19e-13   0.098584   3.92e-13))
   (((2 4) (1 3 5 6 7)) (4.7e-7     3.16e-8    0.000888   3.88e-08))
   (((2 5) (1 3 4 6 7)) (2.23e-7    3.67e-9    9.87e-9    9.79e-11))
   (((2 6) (1 3 4 5 7)) (5.22e-12   1.36e-9    9.87e-9    7.99e-12))
   (((2 7) (1 3 4 5 6)) (4.27e-8    4.4e-10    9.88e-8    3.88e-8))
   (((3 4) (1 2 5 6 7)) (1.7e-7     2.84e-9    0.009877   7.57e-7))
   (((3 5) (1 2 4 6 7)) (4.43e-8    7.3e-10    1.1e-7     1.95e-11))
   (((3 6) (1 2 4 5 7)) (1.04e-12   2.71e-10   1.1e-7     1.59e-12))
   (((3 7) (1 2 4 5 6)) (8.51e-9    8.77e-11   1.1e-6     7.72e-9))
   (((4 5) (1 2 3 6 7)) (0.000399   7.22e-5    9.88e-10   1.93e-6))
   (((4 6) (1 2 3 5 7)) (9.35e-9    2.68e-5    9.88e-10   1.57e-7))
   (((4 7) (1 2 3 5 6)) (7.65e-5    8.68e-6    9.89e-9    0.000764))
```

```
(((5 6) (1 2 3 4 7)) (4.43e-9   3.11e-6    1.1e-14    3.98e-10))
(((5 7) (1 2 3 4 6)) (3.62e-5   1.01e-6    1.1e-13    1.93e-6))
(((6 7) (1 2 3 4 5)) (8.5e-10   3.74e-7    1.1e-13    1.57e-7))
(((1 2 3) (4 5 6 7)) (5.22e-8   3.19e-10   9.87e-5    3.92e-10))
(((1 2 4) (3 5 6 7)) (0.000469  3.15e-5    8.89e-7    3.87e-5))
(((1 2 5) (3 4 6 7)) (0.000222  3.66e-6    9.88e-12   9.78e-8))
(((1 2 6) (3 4 5 7)) (5.21e-9   1.36e-6    9.88e-12   7.98e-9))
(((1 2 7) (3 4 5 6)) (4.27e-5   4.4e-7     9.89e-11   3.87e-5))
(((1 3 4) (2 5 6 7)) (9.35e-5   6.28e-6    9.88e-6    7.72e-6))
(((1 3 5) (2 4 6 7)) (4.43e-5   7.3e-7     1.1e-10    1.95e-8))
(((1 3 6) (2 4 5 7)) (1.04e-9   2.71e-7    1.1e-10    1.59e-9))
(((1 3 7) (2 4 5 6)) (8.5e-6    8.76e-8    1.1e-9     7.72e-6))
(((1 4 5) (2 3 6 7)) (0.398108  0.072174   9.89e-13   0.001927))
(((1 4 6) (2 3 5 7)) (9.34e-6   0.026756   9.89e-13   0.000157))
(((1 4 7) (2 3 5 6)) (0.076405  0.008667   9.9e-12    0.763028))
(((1 5 6) (2 3 4 7)) (4.42e-6   0.003108   1.1e-17    3.97e-7))
(((1 5 7) (2 3 4 6)) (0.036192  0.001007   1.1e-16    0.001927))
(((1 6 7) (2 3 4 5)) (8.49e-7   0.000373   1.1e-16    0.000157))
(((2 3 4) (1 5 6 7)) (4.7e-10   3.16e-11   0.887255   3.88e-11))
(((2 3 5) (1 4 6 7)) (2.23e-10  3.67e-12   9.86e-6    9.8e-14))
(((2 3 6) (1 4 5 7)) (5.23e-15  1.36e-12   9.86e-6    8.0e-15))
(((2 3 7) (1 4 5 6)) (4.27e-11  4.41e-13   9.87e-5    3.88e-11))
(((2 4 5) (1 3 6 7)) (2.0e-6    3.63e-7    8.88e-8    9.69e-9))
(((2 4 6) (1 3 5 7)) (4.7e-11   1.35e-7    8.88e-8    7.91e-10))
(((2 4 7) (1 3 5 6)) (3.84e-7   4.36e-8    8.89e-7    3.84e-6))
(((2 5 6) (1 3 4 7)) (2.23e-11  1.56e-8    9.87e-13   2.0e-12))
(((2 5 7) (1 3 4 6)) (1.82e-7   5.06e-9    9.88e-12   9.69e-9))
(((2 6 7) (1 3 4 5)) (4.27e-12  1.88e-9    9.88e-12   7.91e-10))
(((3 4 5) (1 2 6 7)) (3.99e-7   7.23e-8    9.87e-7    1.93e-9))
(((3 4 6) (1 2 5 7)) (9.36e-12  2.68e-8    9.87e-7    1.58e-10))
(((3 4 7) (1 2 5 6)) (7.66e-8   8.68e-9    9.88e-6    7.65e-7))
(((3 5 6) (1 2 4 7)) (4.43e-12  3.11e-9    1.1e-11    3.98e-13))
(((3 5 7) (1 2 4 6)) (3.63e-8   1.01e-9    1.1e-10    1.93e-9))
(((3 6 7) (1 2 4 5)) (8.51e-13  3.74e-10   1.1e-10    1.58e-10))
(((4 5 6) (1 2 3 7)) (3.99e-8   0.000308   9.88e-14   3.94e-8))
(((4 5 7) (1 2 3 6)) (0.000326  9.98e-5    9.89e-13   0.000191))
(((4 6 7) (1 2 3 5)) (7.65e-9   3.7e-5     9.89e-13   1.56e-5))
(((5 6 7) (1 2 3 4)) (3.62e-9   4.3e-6     1.1e-17    3.94e-8))
(((1 2 3 4) (5 6 7)) (4.7e-7    3.16e-8    0.000888   3.88e-8))
(((1 2 3 5) (4 6 7)) (2.23e-7   3.67e-9    9.87e-9    9.79e-11))
(((1 2 3 6) (4 5 7)) (5.22e-12  1.36e-9    9.87e-9    7.99e-12))
(((1 2 3 7) (4 5 6)) (4.27e-8   4.4e-10    9.88e-8    3.88e-8))
(((1 2 4 5) (3 6 7)) (0.002001  0.000363   8.89e-11   9.68e-6))
(((1 2 4 6) (3 5 7)) (4.69e-8   0.000134   8.89e-11   7.9e-7))
(((1 2 4 7) (3 5 6)) (0.000384  4.36e-5    8.9e-10    0.003834))
(((1 2 5 6) (3 4 7)) (2.22e-8   1.56e-5    9.88e-16   2.0e-9))
(((1 2 5 7) (3 4 6)) (0.000182  5.06e-6    9.89e-15   9.68e-6))
(((1 2 6 7) (3 4 5)) (4.27e-9   1.88e-6    9.89e-15   7.9e-7))
(((1 3 4 5) (2 6 7)) (0.000399  7.22e-5    9.88e-10   1.93e-6))
(((1 3 4 6) (2 5 7)) (9.35e-9   2.68e-5    9.88e-10   1.57e-7))
(((1 3 4 7) (2 5 6)) (7.65e-5   8.68e-6    9.89e-9    0.000764))
```

```
(((1 3 5 6) (2 4 7)) (4.43e-9   3.11e-6    1.1e-14    3.98e-10))
(((1 3 5 7) (2 4 6)) (3.62e-5   1.01e-6    1.1e-13    1.93e-6))
(((1 3 6 7) (2 4 5)) (8.5e-10   3.74e-7    1.1e-13    1.57e-7))
(((1 4 5 6) (2 3 7)) (3.98e-5   0.307689   9.89e-17   3.93e-5))
(((1 4 5 7) (2 3 6)) (0.325725  0.099669   9.9e-16    0.190757))
(((1 4 6 7) (2 3 5)) (7.64e-6   0.036948   9.9e-16    0.015572))
(((1 5 6 7) (2 3 4)) (3.62e-6   0.004292   1.1e-20    3.93e-5))
(((2 3 4 5) (1 6 7)) (2.0e-9    3.63e-10   8.87e-5    9.7e-12))
(((2 3 4 6) (1 5 7)) (4.7e-14   1.35e-10   8.87e-5    7.92e-13))
(((2 3 4 7) (1 5 6)) (3.85e-10  4.36e-11   0.000888   3.84e-09))
(((2 3 5 6) (1 4 7)) (2.23e-14  1.56e-11   9.86e-10   2.0e-15))
(((2 3 5 7) (1 4 6)) (1.82e-10  5.07e-12   9.87e-9    9.7e-12))
(((2 3 6 7) (1 4 5)) (4.28e-15  1.88e-12   9.87e-9    7.92e-13))
(((2 4 5 6) (1 3 7)) (2.0e-10   1.55e-6    8.88e-12   1.98e-10))
(((2 4 5 7) (1 3 6)) (1.64e-6   5.01e-7    8.89e-11   9.6e-7))
(((2 4 6 7) (1 3 5)) (3.84e-11  1.86e-7    8.89e-11   7.83e-8))
(((2 5 6 7) (1 3 4)) (1.82e-11  2.16e-8    9.88e-16   1.98e-10))
(((3 4 5 6) (1 2 7)) (3.99e-11  3.08e-7    9.87e-11   3.94e-11))
(((3 4 5 7) (1 2 6)) (3.26e-7   9.99e-8    9.88e-10   1.91e-7))
(((3 4 6 7) (1 2 5)) (7.66e-12  3.7e-8     9.88e-10   1.56e-8))
(((3 5 6 7) (1 2 4)) (3.63e-12  4.3e-9     1.1e-14    3.94e-11))
(((4 5 6 7) (1 2 3)) (3.26e-8   0.000425   9.89e-17   3.9e-6))
(((1 2 3 4 5) (6 7)) (2.0e-6    3.63e-7    8.88e-8    9.69e-9))
(((1 2 3 4 6) (5 7)) (4.7e-11   1.35e-7    8.88e-8    7.91e-10))
(((1 2 3 4 7) (5 6)) (3.84e-7   4.36e-8    8.89e-7    3.84e-6))
(((1 2 3 5 6) (4 7)) (2.23e-11  1.56e-8    9.87e-13   2.0e-12))
(((1 2 3 5 7) (4 6)) (1.82e-7   5.06e-9    9.88e-12   9.69e-9))
(((1 2 3 6 7) (4 5)) (4.27e-12  1.88e-9    9.88e-12   7.91e-10))
(((1 2 4 5 6) (3 7)) (2.0e-7    0.001546   8.89e-15   1.98e-7))
(((1 2 4 5 7) (3 6)) (0.001637  0.000501   8.9e-14    0.000959))
(((1 2 4 6 7) (3 5)) (3.84e-8   0.000186   8.9e-14    7.83e-5))
(((1 2 5 6 7) (3 4)) (4.04e-12  3.01e-5    9.89e-20   3.99e-9))
(((1 3 4 5 6) (2 7)) (3.99e-8   0.000308   9.88e-14   3.94e-8))
(((1 3 4 5 7) (2 6)) (0.000326  9.98e-5    9.89e-13   0.000191))
(((1 3 4 6 7) (2 5)) (7.65e-9   3.7e-5     9.89e-13   1.56e-5))
(((1 3 5 6 7) (2 4)) (3.62e-9   4.3e-6     1.1e-17    3.94e-8))
(((1 4 5 6 7) (2 3)) (3.26e-5   0.424903   9.9e-20    0.003893))
(((2 3 4 5 6) (1 7)) (2.0e-13   1.55e-9    8.87e-9    1.98e-13))
(((2 3 4 5 7) (1 6)) (1.64e-9   5.02e-10   8.88e-8    9.6e-10))
(((2 3 4 6 7) (1 5)) (3.85e-14  1.86e-10   8.88e-8    7.84e-11))
(((2 3 5 6 7) (1 4)) (1.82e-14  2.16e-11   9.87e-13   1.98e-13))
(((2 4 5 6 7) (1 3)) (1.64e-10  2.14e-6    8.89e-15   1.96e-8))
(((3 4 5 6 7) (1 2)) (3.26e-11  4.26e-7    9.88e-14   3.9e-9))
(((1 2 3 4 5 6) (7)) (2.0e-10   1.55e-6    8.88e-12   1.98e-10))
(((1 2 3 4 5 7) (6)) (1.64e-6   5.01e-7    8.89e-11   9.6e-7))
(((1 2 3 4 6 7) (5)) (3.84e-11  1.86e-7    8.89e-11   7.83e-8))
(((1 2 3 5 6 7) (4)) (1.82e-11  2.16e-8    9.88e-16   1.98e-10))
(((1 2 4 5 6 7) (3)) (1.64e-7   0.002135   8.9e-18    1.96e-5))
(((1 3 4 5 6 7) (2)) (3.26e-8   0.000425   9.89e-17   3.9e-6))
(((2 3 4 5 6 7) (1)) (1.64e-13  2.14e-9    8.88e-12   1.96e-11))
(((1 2 3 4 5 6 7)()) (1.64e-10  2.14e-6    8.89e-15   1.96e-8)) ))
```

Appendix E. *Quattro Data*

The following data was generated using the commercial software spreadsheet Quattro^{TM}. The P(E|H) matrix and the P(-E|H) matrix were developed from inputs provided by the expert. The P({E}|H) is a combination of both the P(E|H) and P(-E|H) matrices and is the knowledge base accessed by the decision-analytic reasoner.

	P(E|H)				P(-E|H)			
1	0.999	0.999	0.001	0.999	0.001	0.001	0.999	0.001
2	0.005	0.005	0.989	0.005	0.995	0.995	0.011	0.995
3	0.001	0.001	0.999	0.001	0.999	0.999	0.001	0.999
4	0.9	0.99	0.9	0.99	0.1	0.01	0.1	0.01
5	0.81	0.92	0.0001	0.2	0.19	0.08	0.9999	0.8
6	0.0001	0.81	0.0001	0.02	0.9999	0.19	0.9999	0.98
7	0.45	0.58	0.001	0.99	0.55	0.42	0.999	0.01
12	0.004995	0.004995	0.000989	0.004995	0.000995	0.000995	0.010989	0.000995
13	0.000999	0.000999	0.000999	0.000999	0.000999	0.000999	0.000999	0.000999
14	0.8991	0.98901	0.0009	0.98901	0.0001	1E-05	0.0999	1E-05
15	0.80919	0.91908	1E-07	0.1998	0.00019	8E-05	0.9989	0.0008
16	9.99E-05	0.80919	1E-07	0.01998	0.001	0.00019	0.9989	0.00098
17	0.44955	0.57942	1E-06	0.98901	0.00055	0.00042	0.998001	1E-05
23	5E-06	5E-06	0.988011	5E-06	0.994005	0.994005	1.1E-05	0.994005
24	0.0045	0.00495	0.8901	0.00495	0.0995	0.00995	0.0011	0.00995
25	0.00405	0.0046	9.89E-05	0.001	0.18905	0.0796	0.010999	0.796
26	5E-07	0.00405	9.89E-05	0.0001	0.9949	0.18905	0.010999	0.9751
27	0.00225	0.0029	0.000989	0.00495	0.54725	0.4179	0.010989	0.00995
34	0.0009	0.00099	0.8991	0.00099	0.0999	0.00999	0.0001	0.00999
35	0.00081	0.00092	9.99E-05	0.0002	0.18981	0.07992	0.001	0.7992
36	1E-07	0.00081	9.99E-05	2E-05	0.9989	0.18981	0.001	0.97902
37	0.00045	0.00058	0.000999	0.00099	0.54945	0.41958	0.000999	0.00999
45	0.729	0.9108	9E-05	0.198	0.019	0.0008	0.09999	0.008
46	9E-05	0.8019	9E-05	0.0198	0.09999	0.0019	0.09999	0.0098
47	0.405	0.5742	0.0009	0.9801	0.055	0.0042	0.0999	0.0001
56	8.1E-05	0.7452	1E-08	0.004	0.189981	0.0152	0.9998	0.784
57	0.3645	0.5336	1E-07	0.198	0.1045	0.0336	0.9989	0.008
67	4.5E-05	0.4698	1E-07	0.0198	0.549945	0.0798	0.9989	0.0098
123	5E-06	5E-06	0.000988	5E-06	0.000994	0.000994	1.1E-05	0.000994
124	0.004496	0.004945	0.00089	0.004945	9.95E-05	9.95E-06	0.001099	9.95E-06
125	0.004046	0.004595	9.89E-08	0.000999	0.000189	7.96E-05	0.010988	0.000796
126	5E-07	0.004046	9.89E-08	9.99E-05	0.000995	0.000189	0.010988	0.000975
127	0.002248	0.002897	9.89E-07	0.004945	0.000547	0.000418	0.010978	9.95E-06
134	0.000899	0.000989	0.000899	0.000989	9.99E-05	9.99E-06	9.99E-05	9.99E-06

135	0.000809	0.000919	9.99E-08	0.0002	0.00019	7.99E-05	0.000999	0.000799
136	9.99E-08	0.000809	9.99E-08	2E-05	0.000999	0.00019	0.000999	0.000979
137	0.00045	0.000579	9.99E-07	0.000989	0.000549	0.00042	0.000998	9.99E-06
145	0.728271	0.909889	9E-08	0.197802	1.9E-05	8E-07	0.09989	8E-06
146	8.99E-05	0.801098	9E-08	0.01978	1E-04	1.9E-06	0.09989	9.8E-06
147	0.404595	0.573026	9E-07	0.97912	5.5E-05	4.2E-06	0.0998	1E-07
156	8.09E-05	0.744455	1E-11	0.003996	0.00019	1.52E-05	0.9988	0.000784
157	0.364136	0.533066	1E-10	0.197802	0.000105	3.36E-05	0.997901	8E-06
167	4.5E-05	0.46933	1E-10	0.01978	0.00055	7.98E-05	0.997901	9.8E-06
234	4.5E-06	4.95E-06	0.88921	4.95E-06	0.0994	0.00994	1.1E-06	0.00994
235	4.05E-06	4.6E-06	9.88E-05	1E-06	0.188861	0.07952	1.1E-05	0.795204
236	5E-10	4.05E-06	9.88E-05	1E-07	0.993906	0.188861	1.1E-05	0.974125
237	2.25E-06	2.9E-06	0.000988	4.95E-06	0.546703	0.417482	1.1E-05	0.00994
245	0.003645	0.004554	8.9E-05	0.00099	0.018905	0.000796	0.0011	0.00796
246	4.5E-07	0.00401	8.9E-05	9.9E-05	0.09949	0.001891	0.0011	0.009751
247	0.002025	0.002871	0.00089	0.004901	0.054725	0.004179	0.001099	9.95E-05
256	4.05E-07	0.003726	9.89E-09	2E-05	0.189031	0.015124	0.010998	0.78008
257	0.001823	0.002668	9.89E-08	0.00099	0.103977	0.033432	0.010988	0.00796
267	2.25E-07	0.002349	9.89E-08	9.9E-05	0.547195	0.079401	0.010988	0.009751
345	0.000729	0.000911	8.99E-05	0.000198	0.018981	0.000799	1E-04	0.007992
346	9E-08	0.000802	8.99E-05	1.98E-05	0.09989	0.001898	1E-04	0.00979
347	0.000405	0.000574	0.000899	0.00098	0.054945	0.004196	9.99E-05	9.99E-05
356	8.1E-08	0.000745	9.99E-09	4E-06	0.189791	0.015185	0.001	0.783216
357	0.000365	0.000534	9.99E-08	0.000198	0.104395	0.033566	0.000999	0.007992
367	4.5E-08	0.00047	9.99E-08	1.98E-05	0.549395	0.07972	0.000999	0.00979
456	7.29E-05	0.737748	9E-09	0.00396	0.018998	0.000152	0.09998	0.00784
457	0.32805	0.528264	9E-08	0.19602	0.01045	0.000336	0.09989	8E-05
467	4.05E-05	0.465102	9E-08	0.019602	0.054994	0.000798	0.09989	9.8E-05
567	3.65E-05	0.432216	1E-11	0.00396	0.10449	0.006384	0.9988	0.00784
1234	4.5E-06	4.95E-06	0.000889	4.95E-06	9.94E-05	9.94E-06	1.1E-06	9.94E-06
1235	4.05E-06	4.6E-06	9.88E-08	9.99E-07	0.000189	7.95E-05	1.1E-05	0.000795
1236	5E-10	4.05E-06	9.88E-08	9.99E-08	0.000994	0.000189	1.1E-05	0.000974
1237	2.25E-06	2.9E-06	9.88E-07	4.95E-06	0.000547	0.000417	1.1E-05	9.94E-06
1245	0.003641	0.004549	8.9E-08	0.000989	1.89E-05	7.96E-07	0.001099	7.96E-06
1246	4.5E-07	0.004005	8.9E-08	9.89E-05	9.95E-05	1.89E-06	0.001099	9.75E-06
1247	0.002023	0.002868	8.9E-07	0.004896	5.47E-05	4.18E-06	0.001098	9.95E-08
1256	4.05E-07	0.003722	9.89E-12	2E-05	0.000189	1.51E-05	0.010987	0.00078
1257	0.001821	0.002665	9.89E-11	0.000989	0.000104	3.34E-05	0.010977	7.96E-06
1267	2.25E-07	0.002347	9.89E-11	9.89E-05	0.000547	7.94E-05	0.010977	9.75E-06
1345	0.000728	0.00091	8.99E-08	0.000198	1.9E-05	7.99E-07	9.99E-05	7.99E-06
1346	8.99E-08	0.000801	8.99E-08	1.98E-05	9.99E-05	1.9E-06	9.99E-05	9.79E-06
1347	0.000405	0.000574	8.99E-07	0.000979	5.49E-05	4.2E-06	9.98E-05	9.99E-08
1356	8.09E-08	0.000744	9.99E-12	4E-06	0.00019	1.52E-05	0.000999	0.000783
1357	0.000364	0.000533	9.99E-11	0.000198	0.000104	3.36E-05	0.000998	7.99E-06
1367	4.5E-08	0.000469	9.99E-11	1.98E-05	0.000549	7.97E-05	0.000998	9.79E-06
1456	7.28E-05	0.73701	9E-12	0.003956	1.9E-05	1.52E-07	0.09988	7.84E-06
1457	0.327722	0.527736	9E-11	0.195824	1.05E-05	3.36E-07	0.09979	8E-08
1467	4.05E-05	0.464637	9E-11	0.019582	5.5E-05	7.98E-07	0.09979	9.8E-08
1567	3.64E-05	0.431784	1E-14	0.003956	0.000104	6.38E-06	0.997801	7.84E-06
2345	3.65E-06	4.55E-06	8.89E-05	9.9E-07	0.018866	0.000795	1.1E-06	0.007952
2346	4.5E-10	4.01E-06	8.89E-05	9.9E-08	0.099391	0.001889	1.1E-06	0.009741

2347	2.03E-06	2.87E-06	0.000889	4.9E-06	0.05467	0.004175	1.1E-06	9.94E-05
2356	4.05E-10	3.73E-06	9.88E-09	2E-08	0.188842	0.015109	1.1E-05	0.7793
2357	1.82E-06	2.67E-06	9.88E-08	9.9E-07	0.103874	0.033399	1.1E-05	0.007952
2367	2.25E-10	2.35E-06	9.88E-08	9.9E-08	0.546648	0.079322	1.1E-05	0.009741
2456	3.65E-07	0.003689	8.9E-09	1.98E-05	0.018903	0.000151	0.0011	0.007801
2457	0.00164	0.002641	8.9E-08	0.00098	0.010398	0.000334	0.001099	7.96E-05
2467	2.03E-07	0.002326	8.9E-08	9.8E-05	0.05472	0.000794	0.001099	9.75E-05
2567	1.82E-07	0.002161	9.89E-12	1.98E-05	0.103967	0.006352	0.010987	0.007801
3456	7.29E-08	0.000738	8.99E-09	3.96E-06	0.018979	0.000152	1E-04	0.007832
3457	0.000328	0.000528	8.99E-08	0.000196	0.01044	0.000336	9.99E-05	7.99E-05
3467	4.05E-08	0.000465	8.99E-08	1.96E-05	0.05494	0.000797	9.99E-05	9.79E-05
3567	3.65E-08	0.000432	9.99E-12	3.96E-06	0.104385	0.006378	0.000999	0.007832
4567	3.28E-05	0.427894	9E-12	0.00392	0.010449	6.38E-05	0.09988	7.84E-05
12345	3.64E-06	4.55E-06	8.89E-08	9.89E-07	1.89E-05	7.95E-07	1.1E-06	7.95E-06
12346	4.5E-10	4.01E-06	8.89E-08	9.89E-08	9.94E-05	1.89E-06	1.1E-06	9.74E-06
12347	2.02E-06	2.87E-06	8.89E-07	4.9E-06	5.47E-05	4.17E-06	1.1E-06	9.94E-08
12356	4.05E-10	3.72E-06	9.88E-12	2E-08	0.000189	1.51E-05	1.1E-05	0.000779
12357	1.82E-06	2.67E-06	9.88E-11	9.89E-07	0.000104	3.34E-05	1.1E-05	7.95E-06
12367	2.25E-10	2.35E-06	9.88E-11	9.89E-08	0.000547	7.93E-05	1.1E-05	9.74E-06
12456	3.64E-07	0.003685	8.9E-12	1.98E-05	1.89E-05	1.51E-07	0.001099	7.8E-06
12457	0.001639	0.002639	8.9E-11	0.000979	1.04E-05	3.34E-07	0.001098	7.96E-08
12467	2.02E-07	0.002323	8.9E-11	9.79E-05	5.47E-05	7.94E-07	0.001098	9.75E-08
12567	4.05E-11	0.003015	9.89E-16	4E-07	0.000189	2.87E-06	0.010986	0.000764
13456	7.28E-08	0.000737	8.99E-12	3.96E-06	1.9E-05	1.52E-07	9.99E-05	7.83E-06
13457	0.000328	0.000528	8.99E-11	0.000196	1.04E-05	3.36E-07	9.98E-05	7.99E-08
13467	4.05E-08	0.000465	8.99E-11	1.96E-05	5.49E-05	7.97E-07	9.98E-05	9.79E-08
13567	3.64E-08	0.000432	9.99E-15	3.96E-06	0.000104	6.38E-06	0.000998	7.83E-06
14567	3.28E-05	0.427466	9E-15	0.003916	1.04E-05	6.38E-08	0.09978	7.84E-08
23456	3.65E-10	3.69E-06	8.89E-09	1.98E-08	0.018884	0.000151	1.1E-06	0.007793
23457	1.64E-06	2.64E-06	8.89E-08	9.8E-07	0.010387	0.000334	1.1E-06	7.95E-05
23467	2.03E-10	2.33E-06	8.89E-08	9.8E-08	0.054665	0.000793	1.1E-06	9.74E-05
23567	1.82E-10	2.16E-06	9.88E-12	1.98E-08	0.103863	0.006346	1.1E-05	0.007793
24567	1.64E-07	0.002139	8.9E-12	1.96E-05	0.010397	6.35E-05	0.001099	7.8E-05
34567	3.28E-08	0.000428	8.99E-12	3.92E-06	0.010439	6.38E-05	9.99E-05	7.83E-05
123456	3.64E-10	3.69E-06	8.89E-12	1.98E-08	1.89E-05	1.51E-07	1.1E-06	7.79E-06
123457	1.64E-06	2.64E-06	8.89E-11	9.79E-07	1.04E-05	3.34E-07	1.1E-06	7.95E-08
123467	2.02E-10	2.32E-06	8.89E-11	9.79E-08	5.47E-05	7.93E-07	1.1E-06	9.74E-08
123567	1.82E-10	2.16E-06	9.88E-15	1.98E-08	0.000104	6.35E-06	1.1E-05	7.79E-06
124567	1.64E-07	0.002137	8.9E-15	1.96E-05	1.04E-05	6.35E-08	0.001098	7.8E-08
134567	3.28E-08	0.000427	8.99E-15	3.92E-06	1.04E-05	6.38E-08	9.98E-05	7.83E-08
234567	1.64E-10	2.14E-06	8.89E-12	1.96E-08	0.010386	6.35E-05	1.1E-06	7.79E-05
1234567	1.64E-10	2.14E-06	8.89E-15	1.96E-08	1.04E-05	6.35E-08	1.1E-06	7.79E-08

P({E}|H)

1,234567	0.010376	6.34E-05	1.1E-09	7.79E-05
2,134567	5.22E-08	3.19E-10	9.87E-05	3.92E-10
3,124567	1.04E-08	6.35E-11	0.001096	7.8E-11
4,123567	9.35E-05	6.28E-06	9.88E-06	7.72E-06
5,123467	4.43E-05	7.3E-07	1.1E-10	1.95E-08
6,123457	1.04E-09	2.71E-07	1.1E-10	1.59E-09
7,123456	8.5E-06	8.76E-08	1.1E-09	7.72E-06
12,34567	5.21E-05	3.19E-07	9.88E-08	3.91E-07
13,24567	1.04E-05	6.35E-08	1.1E-06	7.79E-08
14,23567	0.093383	0.006276	9.89E-09	0.007707
15,23467	0.044234	0.000729	1.1E-13	1.95E-05
16,23457	1.04E-06	0.00027	1.1E-13	1.59E-06
17,23456	0.008489	8.75E-05	1.1E-12	0.007707
23,14567	5.22E-11	3.19E-13	0.098584	3.92E-13
24,13567	4.7E-07	3.16E-08	0.000888	3.88E-08
25,13467	2.23E-07	3.67E-09	9.87E-09	9.79E-11
26,13457	5.22E-12	1.36E-09	9.87E-09	7.99E-12
27,13456	4.27E-08	4.4E-10	9.88E-08	3.88E-08
34,12567	1.7E-07	2.84E-09	0.009877	7.57E-07
35,12467	4.43E-08	7.3E-10	1.1E-07	1.95E-11
36,12457	1.04E-12	2.71E-10	1.1E-07	1.59E-12
37,12456	8.51E-09	8.77E-11	1.1E-06	7.72E-09
45,12367	0.000399	7.22E-05	9.88E-10	1.93E-06
46,12357	9.35E-09	2.68E-05	9.88E-10	1.57E-07
47,12356	7.65E-05	8.68E-06	9.89E-09	0.000764
56,12347	4.43E-09	3.11E-06	1.1E-14	3.98E-10
57,12346	3.62E-05	1.01E-06	1.1E-13	1.93E-06
67,12345	8.5E-10	3.74E-07	1.1E-13	1.57E-07
123,4567	5.22E-08	3.19E-10	9.87E-05	3.92E-10
124,3567	0.000469	3.15E-05	8.89E-07	3.87E-05
125,3467	0.000222	3.66E-06	9.88E-12	9.78E-08
126,3457	5.21E-09	1.36E-06	9.88E-12	7.98E-09
127,3456	4.27E-05	4.4E-07	9.89E-11	3.87E-05
134,2567	9.35E-05	6.28E-06	9.88E-06	7.72E-06
135,2467	4.43E-05	7.3E-07	1.1E-10	1.95E-08
136,2457	1.04E-09	2.71E-07	1.1E-10	1.59E-09
137,2456	8.5E-06	8.76E-08	1.1E-09	7.72E-06
145,2367	0.398108	0.072174	9.89E-13	0.001927
146,2357	9.34E-06	0.026756	9.89E-13	0.000157
147,2356	0.076405	0.008667	9.9E-12	0.763028
156,2347	4.42E-06	0.003108	1.1E-17	3.97E-07
157,2346	0.036192	0.001007	1.1E-16	0.001927
167,2345	8.49E-07	0.000373	1.1E-16	0.000157
234,1567	4.7E-10	3.16E-11	0.887255	3.88E-11
235,1467	2.23E-10	3.67E-12	9.86E-06	9.8E-14
236,1457	5.23E-15	1.36E-12	9.86E-06	8E-15
237,1456	4.27E-11	4.41E-13	9.87E-05	3.88E-11
245,1367	2E-06	3.63E-07	8.88E-08	9.69E-09
246,1357	4.7E-11	1.35E-07	8.88E-08	7.91E-10

247,1356	3.84E-07	' 36E-08	8.89E-07	3.84E-06
256,1347	2.23E-11	1.56E-08	9.87E-13	2E-12
257,1346	1.82E-07	5.06E-09	9.88E-12	9.69E-09
267,1345	4.27E-12	1.88E-09	9.88E-12	7.91E-10
345,1267	3.99E-07	7.23E-08	9.87E-07	1.93E-09
346,1257	9.36E-12	2.68E-08	9.87E-07	1.58E-10
347,1256	7.66E-08	8.68E-09	9.88E-06	7.65E-07
356,1247	4.43E-12	3.11E-09	1.1E-11	3.98E-13
357,1246	3.63E-08	1.01E-09	1.1E-10	1.93E-09
367,1245	8.51E-13	3.74E-10	1.1E-10	1.58E-10
456,1237	3.99E-08	0.000308	9.88E-14	3.94E-08
457,1236	0.000326	9.98E-05	9.89E-13	0.000191
467,1235	7.65E-09	3.7E-05	9.89E-13	1.56E-05
567,1234	3.62E-09	4.3E-06	1.1E-17	3.94E-08
1234,567	4.7E-07	3.16E-08	0.000888	3.88E-08
1235,456	2.23E-07	3.67E-09	9.87E-09	9.79E-11
1236,457	5.22E-12	1.36E-09	9.87E-09	7.99E-12
1237,456	4.27E-08	4.4E-10	9.88E-08	3.88E-08
1245,367	0.002001	0.000363	8.89E-11	9.68E-06
1246,357	4.69E-08	0.000134	8.89E-11	7.9E-07
1247,356	0.000384	4.36E-05	8.9E-10	0.003834
1256,347	2.22E-08	1.56E-05	9.88E-16	2E-09
1257,346	0.000182	5.06E-06	9.89E-15	9.68E-06
1267,345	4.27E-09	1.88E-06	9.89E-15	7.9E-07
1345,267	0.000399	7.22E-05	9.88E-10	1.93E-06
1346,257	9.35E-09	2.68E-05	9.88E-10	1.57E-07
1347,256	7.65E-05	8.68E-06	9.89E-09	0.000764
1356,247	4.43E-09	3.11E-06	1.1E-14	3.98E-10
1357,246	3.62E-05	1.01E-06	1.1E-13	1.93E-06
1367,245	8.5E-10	3.74E-07	1.1E-13	1.57E-07
1456 237	3.98E-05	0.307689	9.89E-17	3.93E-05
1457,236	0.325725	0.099669	9.9E-16	0.190757
1467,235	7.64E-06	0.036948	9.9E-16	0.015572
1567,234	3.62E-06	0.004292	1.1E-20	3.93E-05
2345,167	2E-09	3.63E-10	8.87E-05	9.7E-12
2346,157	4.7E-14	1.35E-10	8.87E-05	7.92E-13
2347,156	3.85E-10	4.36E-11	0.000888	3.84E-09
2356,147	2.23E-14	1.56E-11	9.86E-10	2E-15
2357,146	1.82E-10	5.07E-12	9.87E-09	9.7E-12
2367,145	4.28E-15	1.88E-12	9.87E-09	7.92E-13
2456,137	2E-10	1.55E-06	8.88E-12	1.98E-10
2457,136	1.64E-06	5.01E-07	8.89E-11	9.6E-07
2467,135	3.84E-11	1.86E-07	8.89E-11	7.83E-08
2567,134	1.82E-11	2.16E-08	9.88E-16	1.98E-10
3456,127	3.99E-11	3.08E-07	9.87E-11	3.94E-11
3457,126	3.26E-07	9.99E-08	9.88E-10	1.91E-07
3467,125	7.66E-12	3.7E-08	9.88E-10	1.56E-08
3567,124	3.63E-12	4.3E-09	1.1E-14	3.94E-11
4567,123	3.26E-08	0.000425	9.89E-17	3.9E-06
12345,67	2E-06	3.63E-07	8.88E-08	9.69E-09
12346,57	4.7E-11	1.35E-07	8.88E-08	7.91E-10

12347,56	3.84E-07	4.36E-08	8.89E-07	3.84E-06
12356,47	2.23E-11	1.56E-08	9.87E-13	2E-12
12357,46	1.82E-07	5.06E-09	9.88E-12	9.69E-09
12367,45	4.27E-12	1.88E-09	9.88E-12	7.91E-10
12456,37	2E-07	0.001546	8.89E-15	1.98E-07
12457,36	0.001637	0.000501	8.9E-14	0.000959
12467,35	3.84E-08	0.000186	8.9E-14	7.83E-05
12567,34	4.04E-12	3.01E-05	9.89E-20	3.99E-09
13456,27	3.99E-08	0.000308	9.88E-14	3.94E-08
13457,26	0.000326	9.98E-05	9.89E-13	0.000191
13467,25	7.65E-09	3.7E-05	9.89E-13	1.56E-05
13567,24	3.62E-09	4.3E-06	1.1E-17	3.94E-08
14567,23	3.26E-05	0.424903	9.9E-20	0.003893
23456,17	2E-13	1.55E-09	8.87E-09	1.98E-13
23457,16	1.64E-09	5.02E-10	8.88E-08	9.6E-10
23467,15	3.85E-14	1.86E-10	8.88E-08	7.84E-11
23567,14	1.82E-14	2.16E-11	9.87E-13	1.98E-13
24567,13	1.64E-10	2.14E-06	8.89E-15	1.96E-08
34567,12	3.26E-11	4.26E-07	9.88E-14	3.9E-09
123456,7	2E-10	1.55E-06	8.88E-12	1.98E-10
123457,6	1.64E-06	5.01E-07	8.89E-11	9.6E-07
123467,5	3.84E-11	1.86E-07	8.89E-11	7.83E-08
123567,4	1.82E-11	2.16E-08	9.88E-16	1.98E-10
124567,3	1.64E-07	0.002135	8.9E-18	1.96E-05
134567,2	3.26E-08	0.000425	9.89E-17	3.9E-06
234567,1	1.64E-13	2.14E-09	8.88E-12	1.96E-11
1234567	1.64E-10	2.14E-06	8.89E-15	1.96E-08

Appendix F. *Computer Output*

The following output testing was done in order to determine if the computer results would match that of the expert. Neither conflicting evidence information nor uncertainty assessments were incorporated in this test. The responses recommended by the DA reasoner are determined from the joint likelihood matrix and the scaled utility table. After the recommended action, the expert expected action is given.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(7 1))((2 1)(3 1)(5 1)(6 1)))
                          ------------------------
14.9160660502193         |0.102468532262818      |
7.62023493               |0.0523485405700168     |
47.7074220910387         |0.327734504744941      |
75.3235620340039         |0.517448422422224      |
                          ------------------------
Do action 4  Expected 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(5 1)(6 1))((2 1)(3 1)(4 1)(7 1)))
                          ------------------------
0.00164951870500002      |0.0177405542122364     |
0.060703116705           |0.652861304023074      |
0.00626584711000004      |0.0673891117473198     |
0.02436162877            |0.26200903001737       |
                          ------------------------
Do action 2  Expected 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(6 1))((2 1)(3 1)(4 1)(5 1)(7 1)))
                          ------------------------
2.2255475021483e-4       |0.026347394230459      |
0.00528862635            |0.62609997426044       |
6.6109466042966e-4       |0.0782644343703208     |
0.00227466014004297      |0.269288197138781      |
                          ------------------------
Do action 2  Expected 2
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1)(3 1))((4 1)(5 1)(6 1)(7 1)))
                         ------------------------
1.978629044814e-4       |0.316803337336471       |
1.005795e-8             |1.61040399923114e-5     |
3.87584931087e-4        |0.620572107700108       |
3.9102776028e-5         |0.0626084509234294      |
                         ------------------------
Do action 3  Expected 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((2 1)(3 1))((1 1)(4 1)(5 1)(6 1)(7 1)))
                         ------------------------
0.192534557101804       |0.312500004362359       |
1.005795e-11            |1.63249105313309e-11    |
0.385069106062731       |0.624999995511378       |
0.038506910950556       |0.0625000001099384      |
                         ------------------------
Do action 3  Expected 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1))((2 1)(3 1)(4 1)(5 1)(6 1)(7 1)))
                         ------------------------
1.0141056196883         |0.660416431719646       |
0.0019988955            |0.00130174156208326     |
0.4100160494966         |0.26701492536799        |
0.10943423222966        |0.0712669013502805      |
                         ------------------------
Do action 1  Expected

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1))((3 1)(4 1)(5 1)(6 1)(7 1)))
                         ------------------------
0.0050922216728         |0.6603890311154         |
1.0048185e-5            |0.00130310728459894     |
0.0020591520098         |0.267042852422297       |
5.4952036928e-4         |0.0712650091777042      |
                         ------------------------
Do action 1  Expected 1
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(3 1))((2 1)(4 1)(5 1)(6 1)(7 1)))
                        ------------------------
0.0010185977966         |0.65888228117107       |
2.0008485e-6            |0.00129425336316078    |
4.152503765e-4          |0.268605642225739      |
1.10099057e-4           |0.0712178232400308     |
                        ------------------------
Do action 1  Expected 1
```

Appendix G. *Computer Output Mixed Data*

The following output shows the results of mixing conflicting subsets of the various Evidences. Some cases also show changes given varying belief estimates.

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(6 1))((2 1)(3 1)))

                      -----------------------
0.511754662901934     |0.018878721949372     |
15.7436534745         |0.580786220348139     |
2.71061319532387      |0.099995010375947     |
8.14146318208038      |0.300340047326542     |
                      -----------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(6 1))((2 1)(3 1)(5 1)(7 1)))

                      -----------------------
0.0128961430019315    |0.0156652592287429    |
0.524077785           |0.6366100590557       |
0.061817955763863     |0.0750917775870532    |
0.224440136440386     |0.272632904128504     |
                      -----------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(7 1))((2 1)(3 1)))

                      -----------------------
49.0024469306193      |0.198464062459614     |
20.63953836           |0.0835918793201551    |
73.8466709663987      |0.299085275065154     |
103.419757907844      |0.418858783155077     |
                      -----------------------
Do action 4
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(7 1))((2 1)(3 1)(5 1)(6 1)))
                          ------------------------
14.9160660502193          |0.102468532262818      |
7.62023493                |0.0523485405700168     |
47.7074220910387          |0.327734504744941      |
75.3235620340039          |0.517448422422224      |
                          ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1))((2 1)))
                          ------------------------
97.3609568474787          |0.292171302733066      |
28.828195564896           |0.0865107711177695     |
94.5251347987174          |0.283661260831941      |
112.518155320848          |0.337656665317224      |
                          ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1)(3 1))((2 1)))
                          ------------------------
0.0974301586403542        |0.292227161699813      |
0.028824975396            |0.0864561945047594     |
0.0945985527207083        |0.283734184037681      |
0.112551866900671         |0.337582459757746      |
                          ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(4 1))((2 1)(3 1)(5 1)(6 1)(7 1)))
                          ------------------------
9.19749405991517          |0.60427767052763       |
0.197829135               |0.0129974238723671     |
4.11172370863034          |0.270141280692198      |
1.71359504386303          |0.112583624907804      |
                          ------------------------
Do action 1
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1))((2 1)(3 1)(4 1)(5 1)(6 1)(7 1)))
                       ------------------------
1.0141056196883        |0.660416431719646      |
0.0019988955           |0.00130174156208326З   |
0.4100160494966        |0.26701492536799       |
0.10943423222966       |0.0712669013502805     |
                       ------------------------
Do action 1

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(5 1)(6 1))((2 1)(3 1)))
                       ------------------------
0.335683196305         |0.014802773053579      |
14.490830310705        |0.63900866891291       |
1.68106598839          |0.0741307238138238     |
6.16946858809          |0.272057834219687      |
                       ------------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((2 1)(3 1)))
                       ------------------------
1.93012435611577       |0.312498215740031      |
1.4659819182225e-4     |2.37350890008769e-53   |
3.85966878381155       |0.624902537739005      |
0.386493466611369      |0.0625755114319628     |
                       ------------------------
Do action 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((2 1)(3 1))((7 1)))
                       ------------------------
1.92792567312465       |0.312519717850631      |
4.1553277705l7e-5      |6.73585024838715e-63   |
3.85542633839863       |0.624970541274767      |
0.385579344761487      |0.062503005024354      |
                       ------------------------
Do action 3
```

Conflicting Data

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1)(3 1)(6 1))((5 1)(7 1)))
                           ------------------------
2.5886666855e-7           |0.0537886365566675    |
2.67091293735e-6          |0.554975911214585     |
7.0056884618e-7           |0.145567767612467     |
1.18231617602e-6          |0.245667684616281     |
                           ------------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(6 1)(7 1))((2 1)(3 1)))
                           ------------------------
0.378580025040002         |0.0218690507470333    |
9.3030500745              |0.537399917388498     |
2.064807827376            |0.11927567270615      |
5.564785565094            |0.321455359158319     |
                           ------------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1)(3 1)(7 1))((6 1)(5 1)))
                           ------------------------
8.1494379424e-5           |0.108404109590433     |
3.87365832e-5             |0.051527538954853     |
2.4787185772e-4           |0.329720015276905     |
3.8366184736e-4           |0.510348336177809     |
                           ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(3 1)(4 1))((2 1)(5 1)(6 1)(7 1)))
                           ------------------------
0.009228344408            |0.60308364992305      |
1.980342e-4               |0.0129417783803189    |
0.00415565492             |0.271577156873519     |
0.001719897488            |0.112397414823112     |
                           ------------------------
Do action 1
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(6 1))((2 1)(3 1)(4 1)(5 1)(7 1)))
                      -------------------------
2.2255475021483e-4   |0.026347394230459       |
0.00528862635        |0.62609997426044        |
6.6109466042966e-4   |0.0782644343703208      |
0.00227466014004297  |0.269288197138781       |
                      -------------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1)(3 1)(4 1)(5 1)(6 1)(7 1)))
                      -------------------------
1.04329425736217e-6  |0.0155724159526856      |
4.1985594e-5         |0.626685260821498       |
5.33419053072434e-6  |0.0796191804269471      |
1.86332216274724e-5  |0.278123142798869       |
                      -------------------------
Do action 2

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 1)(2 1)(3 1)(4 1)(7 1))((5 1)(6 1)))
                      -------------------------
7.675228716e-5       |0.103992054725875       |
3.8349108e-5         |0.0519593967214421      |
2.435437608e-4       |0.329978649997262       |
3.794139906e-4       |0.514069898555421       |
                      -------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .9)(2 .9)(3 .9)(7 .94))((6 1)(5 1)))
                      -------------------------
0.156588949164407    |0.114934825494697       |
0.0687980198176093   |0.050497103686516       |
0.454476232684316    |0.333581308092285       |
0.682551962670711    |0.500986762726502       |
                      -------------------------
Do action 4
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 .9)(2 .9)(3 .9)(7 .9))((5 1)(6 1)))

                         ------------------------
0.160755584221771       |0.120627158087363      |
0.0659511644982434      |0.0494881816049404     |
0.450373548043819       |0.337949573828184      |
0.655584641801207       |0.491935086479512      |
                         ------------------------

Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 .9)(2 .9)(3 .9)(7 .8))((5 1)(6 1)))

                         ------------------------
0.171172171865179       |0.136035617389558      |
0.0588340261998287      |0.0467571509457212     |
0.440116836442576       |0.349773943489843      |
0.588166339627446       |0.467433288174878      |
                         ------------------------

Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 .9)(2 .9)(3 .9)(7 .7))((6 1)(5 1)))

                         ------------------------
0.181588759508587       |0.153380050152326      |
0.0517168879014139      |0.0436829839110496     |
0.429860124841333       |0.363083968881515      |
0.520748037453686       |0.43985299705511       |
                         ------------------------

Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 .9)(2 .9)(3 .9)(7 .6))((5 1)(6 1)))

                         ------------------------
0.192005347151995       |0.17304977811234       |
0.0445997496029991      |0.0401966762235804     |
0.41960341324009        |0.378178413431875      |
0.453329735279926       |0.408575132232205      |
                         ------------------------

Do action 4
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .6)(2 .6)(3 .6)(7 .3))((5 1)(6 1)))
                         ------------------------
1.34897207975795         |0.222126855557463      |
0.237394461581074        |0.0390902718218029     |
2.10897738085835         |0.347272209025955      |
2.37763665627653         |0.39151066359478       |
                         ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 0)(2 0)(3 0)(7 0))((5 1)(6 1)))
                         ------------------------
0.0102469422046          |0.608203688254405      |
2.000350485e-4           |0.0118730106844199     |
0.0045709052965          |0.271304493036459      |
0.001829996545           |0.108618808024717      |
                         ------------------------
Do action 1
[19]
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .9)(2 .9)(3 .9)(7 .4))((5 1)(6 1)))
                         ------------------------
0.212838522438812        |0.22152516286124       |
0.0303654730061696       |0.0316047878737935     |
0.399089990037604        |0.415378165692666      |
0.318493130932405        |0.331491883572301      |
                         ------------------------
Do action 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .8)(2 .8)(3 .8)(7 .8))((5 1)(6 1)))
                         ------------------------
0.531687933495788        |0.127958965979455      |
0.20317468238581         |0.0488971455499438     |
1.4010202101881          |0.337177291636632      |
2.01926120331513         |0.48596659683397       |
                         ------------------------
Do action 4
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .8)(2 .8)(3 .8)(7 .4))((5 1)(6 1)))

                      ------------------------
0.557312108814621    |0.202709568277462      |
0.104862786493507    |0.0381414468522542     |
1.02280742971427     |0.37202287412945       |
1.06433095875803     |0.387126110740834      |
                      ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .8)(2 .8)(3 .8)(7 .3))((5 1)(6 1)))

                      ------------------------
0.563718152644329    |0.235092619113691      |
0.0802848125204318   |0.0334319213501335     |
0.928254234595812    |0.387118488540479      |
0.825598397618751    |0.344306970995696      |
                      ------------------------
Do action 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .7)(2 .7)(3 .7)(7 .7))((5 1)(6 1)))

                      ------------------------
0.989363004696017    |0.13693985305305       |
0.348208271387984    |0.0481962528307573     |
2.4276991947236      |0.336023046550646      |
3.4595291958953      |0.478840847565547      |
                      ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .7)(2 .7)(3 .7)(7 .4))((5 1)(6 1)))

                      ------------------------
0.967278856756406    |0.194290233750628      |
0.204450003429965    |0.0410663777867814     |
1.7555418865292      |0.352622866829263      |
2.05125461754907     |0.412020521633328      |
                      ------------------------
Do action 4
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .1)(2 .7)(3 .7)(7 .2))((6 1)(5 1)))

                              -------------------------
0.787412281009515        |0.304950786790926       |
0.0156500274709916       |0.00606097759163707З     |
1.48871234876899         |0.576551843311908       |
0.290321586199009        |0.112436392305529       |
                              -------------------------

Do action 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .7)(2 .7)(3 .7)(7 .3))((5 1)(6 1)))

                              -------------------------
0.959917474109869        |0.226943331469847       |
0.156530580777293        |0.0370069015687517      |
1.5314894504644          |0.362074165095921       |
1.58182975810033         |0.373975601865481       |
                              -------------------------

Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .6)(2 .6)(3 .6)(7 .3))((5 1)(6 1)))

                              -------------------------
1.34897207975795         |0.222126855557463       |
0.237394461581074        |0.0390902718218029      |
2.10897738085835         |0.347272209025955       |
2.37763665627653         |0.39151066359478        |
                              -------------------------

Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .6)(2 .6)(3 .6)(7 .2))((5 1)(6 1)))

                              -------------------------
1.32256694437218         |0.27053871175885        |
0.164719834664703        |0.0336943941181544      |
1.73869706728891         |0.355660533271926       |
1.66265828467939         |0.34010636085107        |
                              -------------------------

Do action 3
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .5)(2 .5)(3 .5)(7 .2))((5 1)(6 1)))
                          ------------------------
1.62175631687925         |0.268410748367406     |
0.213908547691176        |0.0354031939141498    |
2.06639377383231         |0.342001010560807     |
2.14001056614104         |0.354185047157637     |
                          ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .5)(2 .5)(3 .5)(7 .1))((5 1)(6 1)))
                          ------------------------
1.57551165889084         |0.352794245323212     |
0.119531818347839        |0.0267659952931259    |
1.56163106707075         |0.349686053207861     |
1.2091342940616          |0.270753706175801     |
                          ------------------------
Do action 1

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .5)(2 .5)(3 .5)(7 .15))((5 1)(6 1)))
                          ------------------------
1.59863398788504         |0.304273418735187     |
0.166720183019507        |0.0317324168283286    |
1.81401242045153         |0.34526712492151      |
1.67457243010132         |0.318727039514975     |
                          ------------------------
Do action 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .4)(2 .4)(3 .4)(7 .2))((5 1)(6 1)))
                          ------------------------
1.79172377066321         |0.26686133740206      |
0.246052976727611        |0.0366474048714458    |
2.22943970971684         |0.33205523771785      |
2.44684631461471         |0.364436020008644     |
                          ------------------------
Do action 4
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .4)(2 .4)(3 .4)(7 .15))((5 1)(6 1)))
                         ------------------------
1.76052407363575        |0.303917481021484      |
0.191773540471244       |0.0331056713278724     |
1.92992696233168        |0.33316132947617       |
1.91054544691939        |0.329815518174473      |
                         ------------------------
Do action 3

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .3)(2 .3)(3 .3)(7 .1))((5 1)(6 1)))
                         ------------------------
1.70368008021699        |0.356715460200167      |
0.14027460229385        |0.0293706077171912     |
1.54215289390464        |0.322894999851232      |
1.38991216565286        |0.29101893223141       |
                         ------------------------
Do action 1

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .3)(2 .3)(3 .3)(7 .15))((5 1)(6 1)))
                         ------------------------
1.73887447380986        |0.303642971011808      |
0.195651701795715       |0.0341647800985911     |
1.85445017226709        |0.323824846693397      |
1.93773113366571        |0.338367402196204      |
                         ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

(((1 .2)(2 .2)(3 .2)(7 .15))((5 1)(6 1)))
                         ------------------------
1.47752502828582        |0.303424820415188      |
0.17046377522691        |0.0350064732544851     |
1.54073168638023        |0.316404951725444      |
1.68077247467094        |0.345163754604882      |
                         ------------------------
Do action 4
```

```
Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 .2)(2 .2)(3 .2)(7 .1))((5 1)(6 1)))
                      ------------------------
1.44465888779518      |0.358107647539843      |
0.122215849529166     |0.0302953387382462     |
1.26423380050299      |0.313383177207619      |
1.20303845043113      |0.298213836514292      |
                      ------------------------
Do action 1

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 .1)(2 .1)(3 .1)(7 .1))((5 1)(6 1)))
                      ------------------------
0.89834091742109      |0.35925449111275       |
0.0776603828652307    |0.0310570750867758     |
0.764043631299426     |0.305547816677875      |
0.760524830642083     |0.304140617122599      |
                      ------------------------
Do action 1

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 0)(2 0)(3 0)(7 .6))((5 1)(6 1)))
                      ------------------------
0.013603012014658     |0.142062846827397      |
0.0047042602362       |0.0491288694482784     |
0.030959970628444     |0.32332997724548       |
0.0464862374883364    |0.485478306478845      |
                      ------------------------
Do action 4

Enter evidences observed true and false and
Belief in that evidence reading. i.e. (((1 .8))((3 .6)))

((((1 .34)(2 .34)(3 .34)(7 .12))((5 1)(6 1)))
                      ------------------------
1.7526154382486       |0.332865382293166      |
0.163666368289121     |0.0310843252091476     |
1.72327651457286      |0.327293188968683      |
1.62567961434109      |0.308757103529004      |
                      ------------------------
Do action 1
```

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Vita

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